AU–55

B.Sc. (Part–I) Semester–II Examination

MATHEMATICS

Paper–IV

(Vector Analysis and Solid Geometry)

Time : Three Hours] [Maximum Marks : 60

N.B. :– (1) Question No. **1** is compulsory.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative :

(i) Two non-zero vectors $\bar{a}$ and $\bar{b}$ are orthogonal iff ______.

(a) $\bar{a} \cdot \bar{b} = 0$ (b) $\bar{a} \times \bar{b} = 0$

(c) $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ (d) $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$ 1

(ii) The dot product of any two non-zero vectors is a ______.

(a) Vector (b) Scalar

(c) Both vector and scalar (d) None of these 1

(iii) The equation of rectifying plane is ______.

(a) $(\bar{R} - \bar{r}) \cdot \bar{b} = 0$ (b) $(\bar{R} - \bar{r}) \cdot \bar{t} = 0$

(c) $(\bar{R} - \bar{r}) \cdot \bar{n} = 0$ (d) None of these 1

(iv) A line perpendicular to both $\bar{b}$ and $\bar{n}$ is called ______.

(a) Tangent (b) Binormal

(c) Principal normal (d) None of these 1

(v) A vector $\bar{f}$ is irrotational if ______.

(a) div $\bar{f} = 0$ (b) curl $\bar{f} = 0$

(c) div grad $\bar{f} = 0$ (d) curl grad $\bar{f} = 0$ 1

(vi) If $\bar{r} = x_i + y_j + z_k$ then div $\bar{r}$ is equal to ______.

(a) Zero (b) One

(c) Two (d) Three 1

(vii) The curve of intersection of two spheres is a ______.

(a) Plane (b) Circle

(c) Sphere (d) None of these 1

(viii) The equation $x^2 + y^2 + z^2 + 4x - 6y + 10z - 11 = 0$ represents a sphere with centre $(-2, 3, -5)$ then radius of sphere is ______.

(a) 7 (b) 11

(c) 38 (d) None of these 1

(ix) Every section of a right circular cone by a plane perpendicular to its axis is a ______.

(a) Plane (b) Circle

(c) Sphere (d) Cone 1

(x) The equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represent a cone if ______.

(a) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ (b) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} < d$

(c) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} > d$ (d) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = 0$ 1

UNIT–I

2. (a) Prove that :

$$(\bar{a} \times \bar{b}).\left[(\bar{b} \times \bar{c}) \times (\bar{c} \times \bar{a})\right] = [\bar{a}\ \bar{b}\ \bar{c}]^2$$ 4

(b) Prove that necessary and sufficient condition for $\bar{f}(t)$ to have constant magnitude is $\bar{f} \cdot \frac{d\bar{f}}{dt} = 0$. 3

(c) Find the value of r from the equation $\frac{d^2\bar{r}}{dt^2} = \bar{a}t + \bar{b}$, given that both $\bar{r}$ and $\frac{d\bar{r}}{dt}$ vanish when $t = 0$. 3

3. (p) If $\bar{r} = a \cos t\, j + a \sin t\, j + at \tan \alpha\, k$, then find

$$\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right| \text{ and } \left[\dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}}\right]$$ 4

(q) If $\overline{A} = x^2yz\,\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$ and $\overline{B} = 2z\,\vec{i} + 4\vec{j} - x^2\vec{k}$, then find $\frac{\partial^2}{\partial x \partial y}\left(\overline{A} \times \overline{B}\right)$ at $(1, 0, -2)$ 3

(r) Prove that $\vec{i} \times (\bar{a} \times \vec{i}) + \vec{j} \times (\bar{a} \times \vec{j}) + \vec{k} \times (\bar{a} \times \vec{k}) = 2\bar{a}$. 3

UNIT–II

4. (a) For the curve $\bar{r} = \bar{r}(t)$, prove that

$$K = \frac{\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right|}{|\dot{\bar{r}}|^3} \text{ and } T = \frac{\left[\dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}}\right]}{\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right|^2}.$$ 4

(b) The parametric equations of a cycloid are $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, then show that $\rho^2 = 8ay$. 3

(c) Prove that :

$$(x''')^2 + (y''')^2 + (z''')^2 = \frac{1}{\rho^2\sigma^2} + \frac{1 + \rho'^2}{\rho^4}$$ 3

5. (p) Find the curvature and torsion of the circular helix $x = a \cos\theta$, $y = a \sin\theta$, $z = c\theta$ at any point θ. 4

(q) Show that necessary and sufficient condition that a curve be a straight line is $k = 0$. 3

(r) If the tangent and binormal at a point of a curve make angles θ and ϕ respectively with a fixed direction, then show that

$$\frac{\sin\theta\, d\theta}{\sin\phi\, d\phi} = \frac{-K}{T}$$ 3

UNIT–III

6. (a) If $\vec{F} = (3x^2 + 64)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ then evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$. 4

(b) Find $\nabla\phi$, if $\phi = \frac{1}{2}\log(x^2 + y^2 + z^2)$. 2

(c) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$. 4

7. (p) Verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$.

where C is the boundary of the region R bounded by $y = \sqrt{x}$, $y = x^2$. 5

(q) Find the constants a, b, c so that $\vec{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. 5

UNIT–IV

8. (a) Find the equation of the sphere that passes through the circle

$x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$, $3x - 4y + 5z - 15 = 0$ and cuts the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally. 5

(b) Find the equation of the sphere which passes through the points (1, –3, 4), (1, –5, 2) and (1, –3, 0) and whose centre lies on the plane $x + y + z = 0$. 5

9. (p) Prove that the two spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$

will be orthogonal if $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$ 5

(q) Find the equation of a sphere which passes through origin and intercepts lengths a, b and c on the axes respectively. 5

UNIT–V

10. (a) Find the equation of a right circular cone whose vertex is (α, β, γ), the semivertical angle α and the axis $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$. 5

(b) Find the equation of right circular cone whose vertex is (2, –3, 5), axis makes equal angles with the coordinate axes and semivertical angle is 30°. 5

11. (p) Find the equation of the right circular cylinder whose radius is r and axis the line

$$\frac{x-x'}{\ell} = \frac{y-y'}{m} = \frac{z-z'}{n}$$ 5

(q) Find the equation of right circular cylinder of radius 2 and whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}.$$ 5

www.sgbauonline.com