AS–1376

B.Sc. (Part—I) Semester—II Examination
MATHEMATICS
Paper—IV
(Vector Analysis & Solid Geometry)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose correct alternative of the following :—

(i) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are coplaner iff ______.

(a) $\bar{a} \times (\bar{b} \times \bar{c}) = \bar{0}$ (b) $\bar{a} \cdot (\bar{b} \times \bar{c}) = 0$

(c) $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{0}$ (d) $(\bar{a} + \bar{b}) \times \bar{c} = \bar{0}$ 1

(ii) A vector $\bar{f}$ is irrotational if ______.

(a) div $\bar{f} = 0$ (b) div $\bar{f} \neq 0$

(c) curl $\bar{f} = \bar{0}$ (d) None of these 1

(iii) If $\bar{r} = t\bar{i} + \sin t\,\bar{j} + (t^2 - 1)\bar{k}$, then $\dot{\bar{r}}$ at t = 0 is ______.

(a) (0, 0, 1) (b) (0, 1, 0)

(c) (1, 1, 0) (d) (1, 0, 1) 1

(iv) For any space curve, $\bar{t}' \cdot \bar{b}' =$ ______.

(a) k (b) J

(c) kJ (d) −kJ 1

(v) If $\bar{r} = \bar{r}(t)$ is equation of space curve, then the curvature k is equal to ______.

(a) $\dfrac{[\dot{\bar{r}}\ \ddot{\bar{r}}\ \dddot{\bar{r}}]}{|\dot{\bar{r}} \times \ddot{\bar{r}}|^2}$ (b) $\dfrac{\dot{\bar{r}}}{|\dot{\bar{r}}|}$

(c) $\dfrac{\dot{\bar{r}} \times \ddot{\bar{r}}}{|\dot{\bar{r}} \times \ddot{\bar{r}}|}$ (d) $\dfrac{|\dot{\bar{r}} \times \ddot{\bar{r}}|}{|\dot{\bar{r}}|^3}$ 1

(vi) If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then div. $\bar{r}$ is _______.

(a) 3 (b) −2

(c) 0 (d) −1 1

(vii) A vector $\bar{f}$ is solenoidal if _______.

(a) div. $\bar{f} = 0$ (b) curl $\bar{f} = \bar{0}$

(c) div. grad $\bar{f} = 0$ (d) curl grad $\bar{f} = \bar{0}$ 1

(viii) Every section of right circular cone by a plane perpendicular to its axis is ______.

(a) plane (b) circle

(c) sphere (d) None of these 1

(ix) The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represent a real sphere if ______.

(a) $u^2 + v^2 + w^2 = d$ (b) $u^2 + v^2 + w^2 > d$

(c) $u^2 + v^2 + w^2 < d$ (d) $u^2 + v^2 + w^2 = 0$ 1

(x) Two non-parallel planes intersect in a ______.

(a) plane (b) point

(c) line (d) circle 1

UNIT—I

2. (a) If vectors $\bar{f}$ and $\bar{g}$ are vector functions of t, then prove that

$$\frac{d}{dt}(\bar{f} \circ \bar{g}) = \bar{f} \circ \frac{d\bar{g}}{dt} + \frac{d\bar{f}}{dt} \circ \bar{g}.$$ 3

(b) Prove that $\bar{r} = \bar{a}\, e^{mt} + \bar{b}\, e^{nt}$, where $\bar{a}$, $\bar{b}$ are unit vectors is the solution of

$$\frac{d^2\bar{r}}{dt^2} - (m+n)\frac{d\bar{r}}{dt} + mn\,\bar{r} = 0.$$ 3

(c) If $\bar{f} = 2t^2\vec{i} - t\vec{j} + 2\vec{k}$ and $\bar{g} = 7\vec{i} + t^2\vec{j} - t\vec{k}$, then find $\frac{d}{dt}(\bar{f} \times \bar{g})$. 4

3. (p) Prove that :

$$\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}.$$ 4

(q) If $\bar{a}=t\vec{i}-3\vec{j}+2t\vec{k}$, $\bar{b}=\vec{i}-2\vec{j}+2\vec{k}$ and $\bar{c}=3\vec{i}+t\vec{j}-\vec{k}$, then evaluate $\bar{a}.(\bar{b}\times\bar{c})$. 3

(r) Prove that :

$$(\bar{c}\times\bar{a})\times(\bar{a}\times\bar{b})=[\bar{a}\ \bar{b}\ \bar{c}]\bar{a}.$$ 3

UNIT—II

4. (a) State and prove Frenet-Serret formulae. 1+5

(b) If tangent and binormal at a point of a curve makes angle θ, φ respectively with fixed direction, then show that :

$$\frac{\sin\theta\, d\theta}{\sin\phi\, d\phi}=\frac{-k}{J}.$$ 4

5. (p) Prove that $[\bar{r}'', \bar{r}''', \bar{r}'''']=k^3[kJ'-k'J]$. 3

(q) Show that the necessary and sufficient condition that a curve to be a straight line is $k = 0$. 3

(r) Prove that Darboux vector $\bar{d}$ has fixed direction if and only if k/J is constant. 4

UNIT—III

6. (a) Find the work done in moving a particle along the parabola $y^2 = x$ in the xy plane from (0, 0) to (1, 1) if the force field is given by :

$$\bar{f}=(2x+y-7z)\vec{i}+(7x-2y+2z^2)\vec{j}+(3x-2y+4z^3)\vec{k}.$$ 5

(b) Verify Green's theorem in the plane for,

$$\int_c (xy+y^2)dx+x^2dy.$$

Where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5

7. (p) If $\bar{F}=(3x^2+6y)\vec{i}-14yz\vec{j}+20xz^2\vec{k}$, then evaluate $\int_c \bar{F}.d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t$, $y = t^2$, $z = t^3$. 5

(q) Prove that $r^n\,\bar{r}$ is irrotational. Find the value of n when it is solenoidal. 5

UNIT—IV

8. (a) A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 5

(b) Prove that the two spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$

will be orthogonal if $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$. 5

9. (p) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$, $3x - 4y + 5z - 15 = 0$ and cuts the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally. 5

(q) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$. 5

UNIT—V

10. (a) Find the equation of the right circular cylinder of radius 2 and whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}.$$ 5

(b) Find the equation of the right circular cylinder whose radius is r and axis the line :

$$\frac{x-x'}{l} = \frac{y-y'}{m} = \frac{z-z'}{n}.$$ 5

11. (p) Find the equation of a right circular cone whose vertex is (α, β, γ), the semivertical angle α and the axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$. 5

(q) Find the equation of right circular cone whose vertex is (2, –3, 5), axis makes equal angles with the coordinate axes and semi vertical angle is 30°. 5