B.Sc. (Part—I) Semester—II Examination
MATHEMATICS
(Vector Analysis and Solid Geometry)
Paper—IV

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose correct alternative :

(i) The cross product of any two non-zero vectors is a :

(a) Scalar (b) Vector

(c) Both Scalar and Vector (d) None of these 1

(ii) Two non-zero vectors $\bar{a}$ and $\bar{b}$ are parallel iff :

(a) $\bar{a} \cdot \bar{b} = 0$ (b) $\bar{a} \times \bar{b} = 0$

(c) $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ (d) $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$ 1

(iii) The equation of osculating plane is :

(a) $(R - r) \cdot \bar{t} = 0$ (b) $(R - r) \cdot \bar{b} = 0$

(c) $(R - r) \cdot \bar{n} = 0$ (d) None of these 1

(iv) A line perpendicular to both $\bar{t}$ and $\bar{n}$ is called :

(a) tanget line (b) binormal line

(c) principal normal line (d) None of these 1

(v) A vector $\bar{f}$ is solenoidal if :

(a) div $\bar{f} = 0$ (b) curl $\bar{f} = 0$

(c) div $\bar{f} \neq 0$ (d) curl $\bar{f} \neq 0$ 1

(vi) If $\bar{r} = x_i + y_j + z_k$, then div $\bar{r}$ is equal to :

(a) Zero (b) One

(c) Two (d) Three 1

(vii) A plane section of a sphere is a :

(a) Sphere (b) Circle

(c) Both Sphere and Circle (d) None of these 1

(viii) The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a real sphere if :

(a) $u^2 + v^2 + w^2 = d$ (b) $u^2 + v^2 + w^2 > d$

(c) $u^2 + v^2 + w^2 < d$ (d) $u^2 + v^2 + w^2 = 0$ 1

(ix) In Right Circular Cylinder, the radius of the circle is the radius of the :

(a) Circle (b) Sphere

(c) Cylinder (d) Cone 1

(x) Every section of a right circular cone by a plane perpendicular to its axis is a :

(a) Plane (b) Circle

(c) Sphere (d) Cone 1

UNIT—I

2. (a) Prove that a necessary and sufficient condition that $\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}\times\bar{b})\times\bar{c}$ is $(\bar{a}\times\bar{c})\times\bar{b}=0$. 4

(b) If f and g are functions of x, y, z then prove that $\frac{\partial}{\partial x}(\bar{f}\cdot\bar{g})=\bar{f}\cdot\frac{\partial\bar{g}}{\partial x}+\frac{\partial\bar{f}}{\partial x}\cdot\bar{g}$. 3

(c) If $\bar{r}(t)=5t^2\vec{i}+t\vec{j}-t^3\vec{k}$, then prove that $\int_1^2 \bar{r}\times\frac{d^2\bar{r}}{dt^2}\,dt=-14\vec{i}+75\vec{j}-15\vec{k}$. 3

3. (p) If $\bar{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$, $\bar{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$, $\bar{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$, then prove that $\bar{a}\cdot(\bar{b}\times\bar{c})=\bar{b}\cdot(\bar{c}\times\bar{a})=\bar{c}\cdot(\bar{a}\times\bar{b})$. 4

(q) If $\bar{f}=2t^2\vec{i}-t\vec{j}+2\vec{k}$, $\bar{g}=7\vec{i}+t^2\vec{j}-t\vec{k}$, then find $\frac{d}{dt}(\bar{f}\times\bar{g})$. 3

(r) Prove that :

$$(\bar{a}\times\bar{b})\times(\bar{a}\times\bar{c})\cdot\bar{d}=(\bar{a}\cdot\bar{d})\,[\bar{a},\bar{b},\bar{c}].$$ 3

UNIT—II

4. (a) Show that the Serret-Frenet formulae at a point can be written in the form $\bar{t}'=\bar{d}\times\bar{t}$, $\bar{n}'=\bar{d}\times\bar{n}$, $\bar{b}'=\bar{d}\times\bar{b}$ where $\bar{d}=\tau\bar{t}+k\bar{b}$ is a Darboux's vector. 5

(b) Prove that helices are the only twisted curves whose Darboux's vector has a constant direction. 5

5. (p) State and prove Serret-Frenet formulae. 4

(q) Find the equations of the tangent to the curve $x=3t$, $y=3t^2$, $z=2t^3$ at the point $t=1$. 3

(r) Find the curvature and torsion of the circular helix $x=a\cos\theta$, $y=a\sin\theta$, $z=c\theta$ at any point θ. 3

UNIT—III

6. (a) If $\bar{F}=(3x^2+6y)i-14yzj+20xz^2k$, then evaluate $\int_C \bar{F}\cdot d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x=t$, $y=t^2$, $z=t^3$. 4

(b) If $\bar{r}=xi+yj+zk$ then find :

(i) grad $|\bar{r}|$

(ii) div. $\bar{r}$

(iii) curl $\bar{r}$. 2+2+2

7. (p) Verify Green's theorem in the plane for $\int_C (xy + y^2)\, dx + x^2\, dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 4

(q) If $\bar{f} = x^2z\bar{i} - 2y^3z^2\bar{j} + xy^2z\bar{k}$, then find div $\bar{f}$ and curl $\bar{f}$ at (1, −1, 1). 3

(r) Find the work done in moving a particle once around a circle C in the xy plane of radius 2 and centre (0, 0) and if the force field is given by $f = 3xy\bar{i} - y\bar{j} + 2zx\bar{k}$. 3

UNIT—IV

8. (a) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$. 5

(b) Find the equation to the sphere which passes through the points (0, 0, 0), (0, 1, −1), (−1, 2, 0) and (1, 2, 3). 5

9. (p) Show that the spheres :

$$x^2 + y^2 + z^2 + 2x - 6y - 14z + 1 = 0 \text{ and}$$

$$x^2 + y^2 + z^2 - 4x - 8y + 2z + 5 = 0 \text{ are orthogonal.}$$ 5

(q) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3). 5

UNIT—V

10. (a) Find the equation of right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 5

(b) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$. 5

11. (p) Prove that the equation of a cone with vertex at the origin is homogeneous. 5

(q) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$. 5