B.Sc. Part–I (Semester–II) (CBCS) Examination

MATHEMATICS (DSC–III)

(Ordinary Differential Equations)

Paper–III

Time : 3 Hours] [Maximum Marks : 60

N. B. :— Question No. **1** is compulsory, attempt it once only.

1. Choose correct alternative : 10

(i) The order and degree of differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 2\left(\frac{dy}{dx}\right)^3 + 3y = x^2 - e^{3x}$ is :

(a) Order 1, degree 3 (b) Order 2, degree 3

(c) Order 2, degree 2 (d) Order 2, degree 1

(ii) Integrating factor of the differential equation $\frac{dy}{dx} + y/x = x^2$ is :

(a) x (b) log x

(c) e^x (d) xe^x

(iii) The orthogonal trajectories of the family of semi-cubical parabolas $ay^2 = x^3$ is :

(a) $x^2 + 3y^2 = c$ (b) $2x^3 - y^2 = c$

(c) $2x^2 - 3y^2 = c$ (d) $2x^2 + 3y^2 = c$

(iv) General solution of the D. E. sin(Px – y) = p by using Clairaut's form is :

(a) $y = cx - \sin^{-1}c$ (b) $y = cx + \sin^{-1}c$

(c) $y = cx - \sin c$ (d) $y = cx + \sin c$

(v) The roots of the DE $(D^2 - 4D + 13)^2\, y = 0$ are :

(a) Equal and real (b) Distinct and real

(c) Complex and repeated (d) None of these

(vi) Particular Integral of $\frac{1}{P(D^2)}\sin(ax + b)$ is :

(a) $\frac{1}{P(-a^2)}\sin(ax - b)$ (b) $\frac{1}{P(-a^2)}\sin(ax + b)$

(c) $\frac{1}{P(a^2)}\sin(ax - b)$ (d) $\frac{1}{P(a^2)}\sin(ax + b)$

(vii) Let y_1 and y_2 be any two solutions of the DE $y'' + Py' + 9y = 0$, p, $q \in C^o$. If $w(y_1, y_2, x) = 0$ then

(a) y_1 is linearly dependent and y_2 is linearly independent

(b) y_1 is linearly independent and y_2 is linearly dependent

(c) y_1 and y_2 are linearly independent

(d) y_1 and y_2 are linearly dependent

(viii) Particular solution of the DE $y'' + Py' + Qy = 0$ is $y = e^x$ if

(a) $P + xQ = 0$ (b) $1 + P + Q = 0$

(c) $1 - P + Q = 0$ (d) $m^2 + mP + Q = 0$

(ix) Uranium disintegrates at a rate proportional to the amount present at any instant. If M_1 and $M_1/2$ grams of uranium are present at times T_1 and T_2 respectively, then the half of uranium is :

(a) $\frac{1}{2}(T_2 - T_1)$ (b) $T_2 - T_1$

(c) $\frac{1}{3}(T_2 - T_1)$ (d) $2T_2 - T1$

(x) The temperature of water initially is 100°C and that of surrounding is 20°C. If the water cools down to 60°C in first 20 minutes, then the time required to fall temperature up to 30°C is :

(a) 64 min (b) 62 min

(c) 60 min (d) 58 min

UNIT—I

2. (a) Show that :

$\cos x (\cos x - \sin \alpha \sin y) dx + \cos y (\cos y - \sin \alpha \sin x) dy = 0$ is exact and solve. 6

OR

(b) Solve the DE $x^2y - x^3 \frac{dy}{dx} = y^4 \cos x$. 6

(c) Define primitive of a differential equation. Also, find the DE associated with the primitive $y = A \cos mx + B \sin mx$ where A and B being arbitrary constants. 4

OR

(d) Show that the differential equation $x(x - y)dy + y^2dx = 0$ is homogeneous and then solve. 1+3

UNIT—II

3. (a) Solve $3x^4p^2 - xp - y = 0$. 6

OR

(b) Prove that the system of confocal conics $\frac{x^2}{a^2 + ?} + \frac{y^2}{b^2 + ?} = 1$ is self-orthogonal. 6

(c) Explain Clairaut's equation and solve sin px cos y = cos px sin y + P. 2+2

OR

(d) Solve $P^2 + 2Py \cot x = y^2$. 4

UNIT—III

4. (a) Solve $\frac{d^2y}{dx^2} + a^2y = x \cos ax$. 6

OR

(b) Solve the DE :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right).$$ 6

(c) Solve $(D^3 + 3D^2 + 3D + 1)y = e^{-x}$ 4

OR

(d) Solve $(D^3 - 3D^2 + 9D - 27)y = \cos 3x$. 4

UNIT—IV

5. (a) Solve $y'' - \frac{2}{x^2} y' + \left(1 + \frac{2}{x^2}\right)y = xe^{-x}$. 6

OR

(b) Solve the DE $(1 - x^2)\, y'' - xy' - a^2y = 0$ of which $y = ce^{a \sin^{-1} x}$ is an integral. 6

(c) If y_1 and y_2 are linearly dependent differentiable functions then show that their Wronskian vanishes identically. 4

OR

(d) Solve the DE $y'' + n^2y = \operatorname{cosec} nx$ by using variation of parameters. 4

UNIT—V

6. (a) The equation of an LR circuit is given by $L\frac{dI}{dt} + RI = \sin 10\,t$. If I = 0 at t = 0, find the expression for I in terms of t. 6

OR

(b) A man deposits a sum in a bank at 6 percent Compound interest. The compounding is continuous. How much shall he deposit if he will get Rs. 50000 at the end of 4 years ? 6

(c) If 30% of a radioactive substance disappeared in 10 days, how long will it take for 90% of it to disappear. 4

OR

(d) Water at temperature 100°C cools in 10 minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes. 4