B.Sc. Part-I (Semester-II) Examination

MATHEMATICS

(Differential Equations : Ordinary & Partial)

Paper—III

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Solve it in **ONE** attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The roots of the equation $(D^2 - 4D + 13)^2y = 0$ are : 1

(a) distinct and real (b) real and equal

(c) complex and repeated (d) None of these

(ii) A linear equation of first order is of the form Y' + PY = Q in which ? 1

(a) P is function of Y

(b) P and Q are function of X

(c) P is function of X and Q is function of Y

(d) None of these

(iii) The condition for the partial differential equation f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0 to be compatible is that : 1

(a) $J_{pp} + J_{yq} + PJ_{zp} + q.J_{zq} = 0$ (b) $J_{xp} + J_{yq} + PJ_{zp} + q.J_{zq} = 0$

(c) $J_{xp} + J_{qq} + PJ_{zp} + q.J_{zq} = 0$ (d) None of these

(iv) The D.E. $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 v}{\partial t^2}=0$ is called : 1

(a) Partial differential equation (b) Ordinary differential equation

(c) Total differential equation (d) Linear differential equation

(v) An equation of the form Pp + Qq = R where P, Q, R are the functions of X, Y, Z is called : 1

(a) Lagrange's equation (b) Jacobi's equation

(c) Charpit's equation (d) Clairaut's equation

(vi) The particular solution of DE W" + PW' + QW = 0 is $y = e^x$ iff : 1

(a) P + xQ = 0 (b) 1 + p + q = 0

(c) 1 – P + Q = 0 (d) $m^2 + mP + Q = 0$

(vii) The solution of PDE (D – mD')z = 0 is : 1

(a) z = F(y + mx) (b) z = F'(y – mx)

(c) $z = F(e^{xy})$ (d) None of these

(viii) The general form of PDE of first order is : 1

(a) $F(x, y, z, p) = 0$ (b) $F(x, y, z, q) = 0$

(c) $F(x, y, z, p, q) = 0$ (d) $F(y, z, p, q) = 0$

(ix) The complete integral of $F(x, p) = G(y, q)$ is : 1

(a) $z = \int h(x\ a)dx$ (b) $\int k(y\ a)dy$

(c) $z = \int h(x\ a)dx + \int k(y\ a)dy + b$ (d) None of these

(x) The DE $Mdx + Ndy = 0$ is exact iff : 1

(a) $\frac{\partial M}{\partial x} = \frac{\partial M}{\partial y}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$ (d) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

UNIT—I

2. (a) Show that the D.E. :

$$(\sin x \sin y - x e^y)dy = (e^y + \cos x \cdot \cos y)dx$$

is exact and hence solve it. 5

(b) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. 5

3. (p) Solve the D.E. :

$$(1 + x^2)dy + 2xy\ dx = \cot x\ dx.$$ 5

(q) Solve :

$$xy - \frac{dy}{dx} = y^3 e^{-x^2}.$$ 5

UNIT—II

4. (a) Solve the D.E. $(D^2 - 4)y = e^{2x}$. 5

(b) Solve the D.E. $(x^2D^2 - 3xD - 5)y = x^2 \sin(\log x)$. 5

5. (p) Solve the D.E. $(x^2D^2 - xD + 4)y = \cos(\log x)$. 5

(q) Solve the D.E. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin 2x$. 5

UNIT—III

6. (a) Solve the system of D.E. : $D^2x - 2y = 0$ and $D^2y + 2x = 0$. 5

(b) Solve the D.E. $y'' - y = \frac{2}{1+e^x}$ by variation of parameter. 5

7. (p) Solve $x^2y'' + xy' + 10y = 0$ by changing the independent variable from x to $z = \log x$. 5

(q) Solve the following D.E. by removing the first derivative :

$$x\frac{d}{dx}(x\frac{dy}{dx} - y) - 2x\frac{dy}{dx} + 2y + x^2y = 0.$$ 5

UNIT—IV

8. (a) Solve :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$ 5

(b) Find the complete integral of $z = p^2x + q^2y$. 5

9. (p) Find the general solution of PDE $x^2p + y^2q = (x + y)z$. 5

(q) Solve the PDE $p^2 + q^2 = k^2$. 5

UNIT—V

10. (a) Solve the D.E. $(D^2 + 3DD' + 2D'^2)z = x + y$. 5

(b) Solve by Charpits method $pxy + pq + qy = yz$. 5

11. (p) The PDE $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ where f is homogeneous in x, y, z. Prove this. 5

(q) Find a real function v of x and y, reducing to zero when $y = 0$ and satisfying

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -4\pi(x^2 + y^2).$$ 5