Second Semester B. Sc. (Part – I) Examination

(New Course)

MATHEMATICS

Paper – III

(Differential Equations : Ordinary and Partial)

P. Pages : 7

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **1** is compulsory. Solve it once only.

(2) Attempt **One** question from each unit.

1. Choose the correct alternatives :—

(i) The degree of the D.E. (Differential Equation)

$$\left(\frac{d^2y}{dx^2}\right)^2 + 2\left(\frac{dy}{dx}\right)^3 + 3y = x^2 - e^{3x} \text{ is}$$

(a) 1 (b) 2

(c) 0 (d) None of these

1

(ii) The D.E. Mdx + Ndy = 0 is exact if

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$

(c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (d) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ 1

(iii) The particular integral of $\frac{1}{F(D)} \cdot e^{ax}v$, where V is a function of x, is equal to

(a) $\frac{1}{F(a)} e^{ax}V$ (b) $\frac{1}{F(D)} e^{ax}V$

(c) $e^{ax} \frac{1}{F(D-a)} V$ (d) $e^{ax} \frac{1}{F(D+a)} V$ 1

(iv) The particular solution of a D.E. $y''+Py'+Qy=0$ is $y=x$ if

(a) $P+xQ=0$ (b) $1+P+Q=0$

(c) $1-P+Q=0$ (d) $m^2+mp+Q=0$ 1

(v) If M_1 and M_2 are real and distinct roots of auxilliary equation of the D.E. $y''+Py'+Qy=0$, then its solution is given by

(a) $y = C_1 \operatorname{Cos} m_1 x + C_2 \operatorname{Sin} m_2 x$

(b) $y = (C_1 + C_2 x)\ e^x$

(c) $y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$

(d) None of these 1

(vi) The second order linear D.E. $y''+Py'+Qy=R$ (Where P, Q, R are functions of x alone) is

(q) Solve :

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$ 5

UNIT V

10. (a) Solve by Charpit's method $pxy+pq+qy=yz$ 5

(b) Solve :

$(D-D'-1)(D-D'-2)Z=e^{2x-y}+x$ 5

11. (p) Find a real function V of x and y, reducing to zero, when y = 0 and satisfying

$$\frac{\partial^2 V}{dx^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi\ (x^2+y^2)$$ 5

(q) Solve :

$(D^2-3DD'+2D'^2)Z=e^{2x+3y}+\sin(x-2y)$ 5

(b) Find a particular integral of $y''-2y'+y=2x$ by variation of parameters. 5

7. (p) Solve the D.E.

$$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+(x^2+2)y=e^{\frac{x^2+2x}{2}}$$

by reducing it to the normal form. 5

(q) Solve : $\frac{dx}{dt}+\frac{dy}{dt}-2y=2\cos t-7\sin t$

and $\frac{dx}{dt}-\frac{dy}{dt}+2x=4\cos t-3\sin t$ 5

UNIT IV

8. (a) Find the general integral of the linear PDE $(y+zx)p-(x+yz)q=x^2-y^2$ 5

(b) Solve :—

$x^2p^2+y^2q^2=z^2$ 5

9. (p) Form the partial differential equation by eliminating the arbitrary function from the equation.

$V=\frac{1}{r}[f(r-at)+g(r+at)]$ 5

called as Homogeneous equation if value of R is :

(a) 0 (b) 1

(c) 2 (d) 3 1

(vii) The general form of the first order PDE (Partial Differential equation) is

(a) $F(x, y, p, q)=0$

(b) $F(x, y, z, p, q)=0$

(c) $F(x, z, p, q)=0$

(d) None of these 1

(viii) The general solution of the PDE $F(D, D')Z=0$ is

(a) Z = C.F. + P.I. (b) C.F.

(c) Z = P.I. (d) None of these 1

(ix) An equation of the form $Pp+Qq=R$ (Where P, Q, R are function of (x, y, z) is called :

(a) Lagrange's linear equation.

(b) Charpits Auxilliary equation

(c) Jacobi's Auxilliary equation

(d) None of these. 1

(x) The solution of PDE $(2D'-3)Z=0$ is

(a) $Z = e^{2y/3} F(2x)$

(b) $Z = e^{-2y/3} F(2x)$

(c) $Z = e^{-3y/2} F(2x)$

(d) $Z = e^{3y/2} F(2x)$ 1

UNIT I

2. (a) Solve the D.E. $2\cos x \dfrac{dy}{dx} + 4y\sin x = \sin 2x$ given that $y=0$, when $x=\dfrac{\pi}{3}$. 5

(b) Find the orthogonal trajectory of the family of curves $x^2-y^2=Cx$ 5

3. (p) Solve the D.E.

$$3\frac{dy}{dx} + \frac{2}{x+1}y = \frac{x^3}{y^2}$$ 5

(q) Solve the D.E.

$P^2-5P+6=0$ 5

UNIT II

4. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-5x}$$ 5

(b) Solve the D.E.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = x\sin 2x$$ 5

5. (p) Solve the D.E.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = \sin 3x$$ 5

(q) Solve the differential equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x)$$ 5

UNIT III

6. (a) Solve the D.E.

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = x^5 \text{ by}$$

changing the independent variable from x to z. 5