AS–1375

B.Sc. (Part—I) Semester–II Examination

MATHEMATICS (New)

Paper—III

(Differential Equations : Ordinary & Partial)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory. Solve it in **ONE** attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The DE $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x is known as ________. 1

(a) Exact DE (b) Bernoulli's equation

(c) Linear DE of order one (d) Homogeneous DE of order one.

(ii) The order of the DE $\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}-y\sin x=0$ is ________. 1

(a) 1 (b) 2

(c) 3 (d) 4

(iii) The particular solution of the DE $y'' + Py' + Qy = 0$ is $y = e^x$ if ________. 1

(a) $P + xQ = 0$ (b) $1 + P + Q = 0$

(c) $1 - P + Q = 0$ (d) $m^2 + mP + Q = 0$

(iv) The DE $y'' - 4y' + 4y = 0$ has roots which are ________. 1

(a) real and equal (b) real and different

(c) complex (d) None of these

(v) The integrating factor (IF) of the DE $\frac{dy}{dx}+2xy=x$ is ________. 1

(a) x (b) e^x

(c) e^{x^2} (d) e^{-x}

(vi) The value of $\frac{1}{f(D)}e^{ax}, f(a) \neq 0$ is given by ________. 1

(a) $\frac{1}{f(D+a)}e^{ax}$

(b) $\frac{1}{f(D-a)}e^{ax}$

(c) $\frac{1}{f(a)}e^{ax}$

(d) $\frac{1}{f(-a)}e^{ax}$

(vii) The correct value of $\frac{1}{f(D,D')}e^{ax+by}$ is ________. 1

(a) $\frac{1}{f(-a,-b)}e^{ax+by}$

(b) $\frac{1}{f(a,b)}e^{ax+by}$

(c) $\frac{1}{f(-a^2,-b^2)}e^{ax+by}$

(d) None of these

(viii) In PDE $P_p + Q_q = R$, where P, Q and R are functions of ________. 1

(a) x only

(b) y only

(c) x and y only

(d) x, y and z

(ix) Lagrange's form of the PDE of order one is ________. 1

(a) $P_p + Q_q = R$

(b) $P_p - Q_q = R$

(c) $P_q + Q_p = R$

(d) None of these

(x) The solution of the PDE $r = a^2t$ is ________ . 1

(a) $z = F_1(y + ax) + F_2(y - ax)$

(b) $z = F_1(y - ax) + F_2(y - ax)$

(c) $z = F(y + ax)$

(d) None of these

UNIT—I

2. (a) Show that the DE $(e^y + 1) \cos x\, dx + e^y \sin x\, dy = 0$ is exact and hence solve it. 5

(b) Solve the DE $\cos x\, dy = y(\sin x - y)\, dx$. 5

3. (p) Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2gx + c = 0$, where g is a parameter. 5

(q) Solve the DE $(p-xy)(p-x^2)(p-y^2) = 0$. 5

UNIT—II

4. (a) Solve the DE $\frac{d^2y}{dx^2}+a^2y=x\cos ax$. 5

(b) Solve the DE $(x^2D^2-3xD+5)y=x^2\sin(\log x)$. 5

5. (p) Solve the DE $y''+3y'+2y=4x-20\cos 2x$. 5

(q) Solve the DE $\frac{d^2y}{dx^2}+4y=e^x+\sin 2x$. 5

UNIT—III

6. (a) Find the particular solution of $y''-2y'+y=2x$ by variation of parameters. 5

(b) Solve the DE $\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-y\sin^2 x=\cos x-\cos^3 x$ by changing the independent variable x to z. 5

7. (p) Solve the simultaneous DEs.

$$\frac{dx}{dt}+2\frac{dy}{dt}-2x+2y=3e^t;\ 3\frac{dx}{dt}+\frac{dy}{dt}+2x+y=4e^{2t}$$ 5

(q) Solve the DE $x^2y''-3xy'+3y=(2x+1)x^2$. 5

UNIT—IV

8. (a) Solve the PDE $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$. 5

(b) Form the PDE by eliminating the arbitrary functions from $f(x+y+z, x^2+y^2+z^2)=0$. 5

9. (p) Solve the PDE $p^2+q^2=x^2+y^2$. 5

(q) Solve the PDE

$$\frac{dx}{x(y^2-z^2)}=\frac{dy}{-y(z^2+x^2)}=\frac{dz}{z(x^2+y^2)}.$$ 5

UNIT—V

10. (a) Solve the PDE $r+s-6t=y\cos x$. 5

(b) Solve the PDE $D(D-2D'-3)z=e^{x+2y}$ 5

11. (p) Solve the PDE $r-3s+2t=e^{2x+3y}+\sin(x-2y)$. 5

(q) Solve the PDE $(D^2-2DD'-8D'^2)z=\sqrt{2x+3y}$. 5