AU–54

B.Sc. (Part-I) Semester-II Examination

MATHEMATICS

Paper—III

(Differential Equations : Ordinary & Partial)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory. Solve it in **ONE** attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The form of the D.E. $Y = px + f(p)$ is called : 1

(a) Exact D. Eqⁿ (b) Bernoulli's D. Eqⁿ

(c) Homogenous D. Eqⁿ (d) Clairaut's D. Eqⁿ

(ii) Order and degree of Differential Equation $\left(\frac{d^2y}{dx^2}\right)^2 + 2\left(\frac{dy}{dx}\right)^3 + 3y = x^2 - \rho^{3x}$ is : 1

(a) Order 1 and degree 3 (b) Order 2 and degree 3

(c) Order 2 and degree 2 (d) Order 1 and degree 2

(iii) The D.E. $(D^2 + 9)y = 0$ has roots which are : 1

(a) real and equal (b) real and distinct

(c) imaginary (d) None of these

(iv) Which is the correct result given below : 1

(a) $\frac{1}{P(D^2)} \sin ax = \frac{1}{P(-a^2)} \sin ax, P(-a^2) \neq 0$

(b) $\frac{1}{P(D^2)} \cos ax = \frac{1}{P(-a^2)} \cos ax, P(-a^2) \neq 0$

(c) Both (a) and (b) are correct

(d) None of these

(v) The particular solution of D.E. $W'' + PW' + QW = 0$ is $y = x$ if : 1

(a) $1 + P + Q = 0$ (b) $1 - P + Q = 0$

(c) $M^2 + MP + Q = 0$ (d) $P + xQ = 0$

(vi) $Pdx + Qdy + Rdz = 0$ is form of equation : 1

(a) Pfaffian DE (b) Total D.E.

(c) Both (a) and (b) (d) None of these

(vii) The integrating factor of the DE $\frac{dy}{dx} - xy = x^2$ is : 1

(a) $e^{-x^2/2}$ (b) $e^{x^2/2}$

(c) e^x (d) e^{-x}

(viii) The complete integral of $F(x, p) = G(y, q)$ is : 1

(a) $z = \int h(x, a)\, dx$ (b) $\int k(y, a)\, dy$

(c) $z = \int h(x, a)\, dx + \int k(y, a)\, dy + b$ (d) None of these

(ix) The PDE's $f(x, y, p, q) = 0$ $g(x, y, p, q) = 0$ are compatible if : 1

(a) $J_{xp} + J_{yq} + uJ_{zp} + VJ_{zq} = 0$ (b) $J_{xp} + J_{yq} + VJ_{zp} = 0$

(c) $J_{xp} + J_{yq} = 0$ (d) $uJ_{xp} + VJ_{yq} = 0$

(x) Solution of D.E. $p^2 - q^2 = x - y$ is : 1

(a) $z = \frac{2}{3}(x + a)^{3/2} + \frac{2}{3}(y + a)^{3/2} + b$

(b) $z = \frac{2}{3}(x - a)^{3/2} + \frac{2}{3}(y + a)^{3/2} + b$

(c) $z = (x + a)^{3/2} + (y + a)^{3/2} + b$

(d) $z = (x + a)^{3/2} + (y + a)^{2/3}$

UNIT—I

2. (a) Solve the D.E. $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 5

(b) $xdx + ydy = a^2 \frac{xdy - ydx}{x^2 + y^2}$. Show that this D.E. is exact and solve it. 5

3. (p) Solve :

$(p - xy)(p - x^2)(p - y^2) = 0$. 5

(q) Find orthogonal trajectories of the family of semicubical parabolas $ay^2 = x^3$. 5

UNIT—II

4. (a) Solve the D.E. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$. 5

(b) Solve the D.E. $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \log x$. 5

5. (p) Solve the D.E. $y'' + 2y' + y = e^{-x} \cdot \log x$. 5

(q) Solve D.E. $(x^2D^2 - xD + 4)y = \cos(\log x)$. 5

UNIT—III

6. (a) Solve the D.E. $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 10y = 0$ by changing the independent variable from x to $z = \log x$. 5

(b) Solve $y'' + n^2y = \text{cosec } nx$ by variation of parameter. 5

7. (p) Solve the D.E. $(x \sin x + \cos x)y'' - x \cos x\, y' + y \cos x = 0$. 5

(q) Solve Simultaneous equation $D^2x - 2y = 0$ and $D^2y + 2x = 0$. 5

UNIT—IV

8. (a) Solve :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$ 5

(b) Find the general solution of P.D.E. $x^2p + y^2q = (x + y)z$. 5

9. (p) Solve P.D.E. $p^2 + q^2 = k^2$. 5

(q) Find the complete integral of $z = p^2x + q^2y$. 5

UNIT—V

10. (a) Find the complete solution of $(p^2 + q^2)y = qz$. 5

(b) Show that the P.D.E. $z = px + qy$ is compatible with any equation $f(xyzpq) = 0$ where f is homogeneous in x, y, z. 5

11. (p) Show that the equations $z = px - qy$ and $2xy(p^2 + q^2) = x(yp + xq)$ are compatible. Hence solve these equations. 5

(q) Solve by Charpit method $pxy + pq + qy = yz$. 5