AP–420

B.Sc. (Part–I) Semester–II Examination

2S : MATHEMATICS

(Integration & Differential Equations)

Paper—III

Time—Three Hours] [Maximum Marks—60

Note :— (1) Question No. **1** is compulsory. Solve it in **one** attempt only.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives :

(i) If $A(t_1)$ and $B(t_2)$ are the points on the curve $x = f(t)$, $y = g(t)$, $t_1 \le t \le t_2$, then the length of arc AB of the curve is given by :

(a) $s = \int_{t_1}^{t_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dt$

(b) $s = \int_{t_1}^{t_2} \sqrt{1 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

(c) $s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

(d) None of these 1

(ii) The volume of right circular cone of height 'h' with base radius 'a' is given by :

(a) $\frac{1}{3}\pi a^2 h^2$ (b) $\frac{1}{3}\pi a^2 h$

(c) $\frac{1}{3}\pi^2 a^2 h$ (d) $\frac{1}{3}\pi a h^2$ 1

(iii) The degree of the DE $\frac{d^3y}{dx^3} = \sqrt[4]{y + \left(\frac{dy}{dx}\right)^5}$ is :

(a) 1 (b) 2

(c) 3 (d) 4 1

(iv) The area enclosed by the curve y = f(x), x-axis and the ordinates x = a and x = b is :

(a) $\int_a^b x^2\,dy$ (b) $\int_a^b y^2\,dx$

(c) $\int_a^b x\,dy$ (d) $\int_a^b y\,dx$ 1

(v) The order of the DE $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} - y\sin x = 0$:

(a) 1 (b) 2

(c) 3 (d) 0 1

UNIT–V

10. (a) Solve $x^2y'' + 3xy' + 10y = 0$ by changing the independent variable from x to z = log x. 5

(b) Solve the DE $y'' - y = \frac{2}{1+e^x}$ by variation of parameters. 5

11. (c) Solve the differential equation :

$$(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx} + y\cos x = 0.$$ 5

(d) Solve the simultaneous differential equations $\frac{dx}{dt} + 7x - y = 0$; $\frac{dy}{dt} + 2x + 5y = 0$. 5

UNIT–III

6. (a) Solve the DE $(x + y)\,dx - (x - y)\,dy = 0$. 3

(b) Solve $\frac{dy}{dx} + \frac{y}{x} = x^2$, for $x = 1$, $y = 1$. 3

(c) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. 4

7. (d) Solve the DE $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$. 3

(e) Solve $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 4

(f) Solve $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$. 3

UNIT–IV

8. (a) Solve $\frac{d^2y}{dx^2} + a^2 y = x \cos ax$. 5

(b) Solve the DE $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\ln x)$. 5

9. (c) Solve the DE $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \sin 2x + e^{2x}$. 5

(d) Solve $(x + a)^2 \frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$. 5

(vi) The DE $\frac{dy}{dx} + Py = Qy^n$ (where P and Q are functions of x alone) is known as :

(a) Linear DE (b) Exact DE

(c) Simultaneous DE (d) Bernoulli's equation 1

(vii) The PI $\frac{1}{f(D)} e^{ax}$, $f(a) \neq 0$ is given by :

(a) $PI = \frac{1}{f(D + a)} \cdot e^{ax}$

(b) $PI = \frac{1}{f(D - a)} \cdot e^{ax}$

(c) $PI = \frac{1}{f(a)} \cdot e^{ax}$

(d) $PI = \frac{1}{f(-a)} e^{ax}$ 1

(viii) The primitive of $\frac{d^2y}{dx^2} + 9y = 0$ is :

(a) $y = c_1 \cos x + c_2 \sin x$

(b) $y = c_1 \cos 3x + c_2 \sin 3x$

(c) $y = (c_1 + c_2) \cos 3x$

(d) None of these 1

(ix) The linear DE $Y'' + PY' + QY = R$, (where P, Q, R are function of x alone) is called as Homogeneous equation if value of R is :

(a) 1 (b) –1

(c) 0 (d) 2 1

(x) The particular solution of a differential equation $Y'' + PY' + QY = 0$ is $Y = e^x$ if :

(a) $P + xQ = 0$ (b) $1 + P + Q = 0$

(c) $1 - P + Q = 0$ (d) $m^2 + mP + Q = 0$ 1

UNIT–I

2. (a) Evaluate $\int \frac{x^3 + 3}{\sqrt{x^2 + 1}}\, dx$. 3

(b) Evaluate $\int \frac{1 + x^{1/2}}{1 + x^{1/3}}\, dx$. 3

(c) If $I_n = \int \cot^n x\, dx$, then prove that :

$$I_n = -\frac{1}{n-1}\cot^{n-1} x - I_{n-2}$$

Hence find $\int \cot^5 x\, dx$. 4

3. (d) Evaluate $\int \frac{dx}{(x+1)\sqrt{1 + 2x - x^2}}$. 3

(e) Prove that $\int_0^1 x^{3/2}(1 - x)^{3/2}\, dx = \frac{3\pi}{128}$. 3

(f) If $I_n = \int \operatorname{cosec}^n x\, dx$, then prove that :

$$I_n = -\frac{1}{n-1}\operatorname{cosec}^{n-2} x \cot x + \frac{n-2}{n-1} I_{n-2}$$

Hence find $\int \operatorname{cosec}^3 x\, dx$. 4

UNIT–II

4. (a) Show that the length of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ from $\theta = 0$ to $\theta = 2\pi$ is 8a. 3

(b) Find the area between the curve $y = x^3 - 3x^2 + 2x$ and the x-axis. 3

(c) Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis. 4

5. (d) Show that the volume of a sphere of radius 'a' is $\frac{4}{3}\pi a^3$. 3

(e) Find the whole area included between the curve $x^2y^2 = a^2(y^2 - x^2)$ and its asymptotes. 3

(f) Find the total length of the curve :

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$ 4