AV–1664

B.Sc. (Part—I) (Semester—I) Examination

MATHEMATICS

(Algebra and Trigonometry)

Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The period of sinh z is :

(a) $2\pi i$ (b) πi

(c) $\frac{\pi}{2} i$ (d) i 1

(ii) The value of $e^{-\frac{\pi}{2}i}$ is :

(a) $-i$ (b) $1 + i$

(c) $1 - i$ (d) 0 1

(iii) The series $4 \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$ is called :

(a) Gregory's series (b) Euler's series

(c) Rutherford's series (d) Machiri's series 1

(iv) The sum of infinite Geometric series $a + ar + ar^2 \ldots\ldots + ar^{n-1} + \ldots\ldots\ldots, |r| < 1$ is :

(a) $\frac{d}{1-r}$ (b) $\frac{r}{1-r}$

(c) $\frac{r}{a-r}$ (d) .1 1

(v) The norm of quaternion $q = 2 + 2\vec{i} - \vec{j} + 4\vec{k}$ is :

(a) 2 (b) 9

(c) 4 (d) 5 1

(vi) For any quaternion q, its inverse is equal to :

(a) $-q$ (b) q^*

(c) $-q^*$ (d) None of these 1

(vii) The polynomial of fourth degree is called as :

(a) Linear (b) Quadratic

(c) Biquadratic (d) Cubic 1

(viii) The degree of an equation having roots $(3 + i)$ is :

(a) 1 (b) 2

(c) 3 (d) 4 1

(ix) The rank of zero matrix is :

(a) 1 (b) 0

(c) n (d) None of these 1

(x) The number of positive and negative roots of an equation of degree n is found by :

(a) Cardan's Method (b) Ferrari's Method

(c) Descartes' rule of signs (d) None of these 1

UNIT—I

2. (a) State DeMoivre's theorem and prove it for positive integer. 1+4

(b) Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right)$, where n being positive integer. 5

3. (p) Separate the following expression into real and imaginary parts :

(i) $\sinh(x + iy)$

(ii) $\tan(x + iy)$. 4

(q) Find all the value of $(-1)^{1/3}$. 3

(r) Show that $\cosh^{-1} x = \log\left\{x + \sqrt{x^2 - 1}\right\}$. 3

UNIT—II

4. (a) If $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$, then prove that :

$$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x \ldots\ldots + (-1)^{n-1}\frac{\tan^{2n-1} x}{2n-1} + \ldots\ldots$$ 5

(b) Sum the series :

$$S = a\sin x + \frac{1}{2}a^2\sin 2x + \frac{1}{3}a^3\sin 3x + \ldots\ldots$$ 5

5. (p) Prove that :

$$\frac{\pi}{4} = \frac{1}{2} - \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^5} + \ldots\ldots + \frac{1}{3} - \frac{1}{3}\cdot\frac{1}{3^3} + \frac{1}{5}\cdot\frac{1}{3^5}\ldots\ldots$$ 5

(q) Sum the series :

$$a\sin x - \frac{1}{3}a^3\sin 3x + \frac{1}{5}a^5\sin 5x - \ldots\ldots$$ 5

UNIT—III

6. (a) If $p = 2 - 3\vec{i} - 4\vec{j} + 5\vec{k}$ and $q = -6 + \vec{i} + 2\vec{j} - 3\vec{k}$, then find the quaternion product pq. 4

(b) Show that :

$pq = qp \Leftrightarrow \overline{p}$ and $\overline{q}$ are parallel, for some $p, q \in H$. 4

(c) Prove that quaternion product $\vec{i}\cdot\vec{j} = \vec{k}$. 2

7. (p) Let q is any unit quaternion, then prove that :

$$Lq(\vec{v}) = \vec{w} = (q_0 - |\vec{q}|^2)\vec{v} + 2(\vec{q} \cdot \vec{v})\vec{q} + 2q_0(\vec{q} \times \vec{v})$$ 5

(q) If the quaternion $q = \cos\frac{\pi}{3} + i \sin\frac{\pi}{3}$ and input vector $\bar{v} = i$, then compute the output vector $\vec{w}$ under the action of operator Lq. 5

UNIT—IV

8. (a) Find the equation whose roots are the roots of equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4. 4

(b) If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in G.P. Prove that $a^3c = b^3$. 3

(c) Find the equation whose roots are the reciprocals of $x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$. 3

9. (p) Solve the equation $x^3 - 21x = 344$ by Cardan's method. 4

(q) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the values of :

(i) $\Sigma\alpha^2$

(ii) $\Sigma\alpha^2\beta$. 4

(r) Show that the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four complex roots. 2

UNIT—V

10. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 5

(b) Verify Cayley-Hamilton theorem for matrix A :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$ 5

11. (p) Find the row rank and column rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$. 5

(q) Show that the eigen values of Hermitian matrix are all real. 5

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