First Semester B. Sc. (Part - I) Examination

MATHEMATICS

Paper - II

(Differential and Integral Calculus)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **One** is compuslsory attempt once.

(2) Attempt **One** question from ech units.

1. Choose the correct alternatives (1 mark each):—

(i) $\lim_{x\to 0} \frac{1}{x} \cos \frac{1}{x} =$ ------

(a) Limit exist.

(b) Limit does not exist.

(c) Equal to zero.

(d) None of these.

(ii) The function f is defined by f(x) = tan x is discontinuous at ————

(a) $x = \frac{\pi}{2}$ only

(b) $x = n\pi$, $\forall n \in \pi$

(c) $x = \frac{n\pi}{2}$, $\forall n \in N$

(d) $x = (2n+1)\frac{\pi}{2}$, $n = 0, 1, 2, ----$

(iii) The modulus function $f(x) = |x|$, $\forall x \in R$ is ———— at $x = 0$

(a) Continuous but not derivable.

(b) Derivable but not continous.

(c) Continuous and derivable.

(d) None of these.

(iv) $\lim_{x \to 0} \frac{\sin x}{x} =$ ------

(a) 0 (b) 1

(c) ∞ (d) None of these

(v) Let f be differentiable function on (a, b). Then which of the following statement is correct :

(a) $f'(x) \geq 0, \forall x \in (a,b) \Rightarrow f$ is monotone decreasing.

(b) $f'(x) = 0, \forall x \in (a,b) \Rightarrow f$ is not constant.

(c) $f'(x) \leq 0, \forall x \in (a,b) \Rightarrow f$ is monotone decreasing

(d) $f'(x) \leq 0 \ \forall x \in (a,b) \Rightarrow f$ is not decreasing.

(vi) The series $f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \text{----} + \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + \text{------}$ is called-------

(a) Taylor's series.

(b) Maclaurin's series.

(c) Lagranges series.

(d) None of these.

(vii) If $u = \frac{x^4 - y^4}{x-y}$, $x \neq y$ then $xu_x + yu_y =$----

(a) 1.u (b) 4u

(c) 3u (d) None of these

(viii) Let $f(x, y) = 2x^3y^2 - 3xy^2 + x - 2y$ then f_{yy} = ————

(a) $4x^3 - 6x$ (b) $12x^2 - 6y$

(c) $4y - 6$ (d) None of these

(ix) The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) πa (b) πab

(c) $\sqrt{\pi}\, ab$ (d) None of these

(x) $\int_0^{\pi/8} \cos^3 4x \, dx =$ ——

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{8}$ 10

UNIT I

2. (a) Let F(x) and g(x) be defined at all points of an interval [a, b] except possibly at $x_0 \in [a, b]$. If $\lim_{x \to x_0} F(x) = l$, $\lim_{x \to x_0} g(x) = m$, then prove that

$$\lim_{x \to x_0} [f(x) + g(x)] = l + m$$

(b) Show that $f(x) = \begin{cases} \dfrac{e^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

has a simple discontinuity at $x = 0$. 3

(c) Prove that $F(x) = x^2$ is continuous at $x = 3$. 3

3. (P) Show that the function $f(x)=\begin{cases}(1+2x)^{1/x} & x\neq 0\\ e^2 & , x=0\end{cases}$

is continuous at $x = 0$ 3

(q) Prove that $\lim_{x\to a} \sin x = \sin a$ by ϵ–δ definition. 4

(r) If a function f is continuous on the closed interval $I = [a, b]$ and $f(a) \neq f(b)$, then f assumes every value between f(a) and f(b). 3

UNIT II

4. (a) Justify, by an example, that continuity of a function at a point necessarily not imply the derivability at that point. 4

(b) If $y = \frac{1}{x^2+a^2}$, then find y_n. 3

(c) Evaluate $\lim_{x\to 0} (\cos x)^{1/x^2}$ 3

5. (p) Find the right hand and left hand derivative of $f(x) = |x|$ at $x = 0$ 3

(q) If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. 4

(r) Evaluate $\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan 5x}{\tan x} \right)$ 3

UNIT III

6. (a) State and prove Rolle's mean value theorem. 4

(b) Verify Cauchy mean value theorem for $f(x) = \cos x$ $g(x) = \sin x$ in $[0, \pi/2]$ 3

(c) Show that :

$$\log(x+h) = \log h + \frac{x}{h} - \frac{x^2}{2h^2} + \frac{x^3}{3h^3} - \text{----}$$

7. (p) Let f be differentiable on (a, b) then Prove that $f(x) \geq 0 \ \forall \ x \in (a, b) \Rightarrow f$ is monotone increasing. 3

(q) Expand $2x^3 + 7x^2 + x - 1$ in power of $(x-2)$. 3

(r) State and prove Lagrange's mean value theorem. 4

UNIT IV

8. (a) If $u = \frac{x^2+y^2}{x+y}$ Prove that

$$\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$$ 3

(b) Let F(u) be a hamogeneous function of degree n in x and y, where u is a function of x, y. Then :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{nF(u)}{F'(u)}$$ and

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = G(u)[G'(u)-1]$$

where $G(u) = \frac{nF(u)}{F'(u)}$ and suitable condition of differentiability. 4

(c) If $u = \log(x^3+y^3+z^3-3xyz)$, show that

$$u_x+u_y+u_z = \frac{3}{x+y+z}$$ 3

9. (p) If $Z = f(xy)$, show that $xz_x - yz_y = 0$

(q) Verify Euler's theorem on homogeneous functions for $3x^2yz + 5xy^2z + 4z^4$. 4

(r) If $u = \dfrac{e^{x+y+z}}{e^x+e^y+e^z}$ then show that

$u_x + u_y + u_z = 2u$ 3

UNIT V

10. (a) If $I_n = \int \sec^n x \, dx$, then prove that

$$I_n = \frac{1}{n-1} \sec^{n-2} x \cdot \tan x + \frac{n-2}{n-1} I_{n-2}$$ 4

(b) Find the area between the curve

$y = x^3 - 3x^2 + 2x$ and the x–axis. 3

(c) Prove that $\int_0^1 x^{3/2}(1-x)^{3/2} \, dx = \dfrac{3\pi}{128}$ 3

11. (p) If $\phi(n) = \int_0^{\pi/4} \tan^n x \, dx$, then show

that $\phi(n) + \phi(n-2) = \dfrac{1}{n-1}$ and find the value of $\phi(5)$ 4

(q) Show that in the catenary $y = C \cosh \frac{x}{c}$, the length of the arc from the vertex to any print is given by $S = C \sinh \frac{x}{a}$. 3

(r) Integrate $\int \dfrac{2x^2 - x + 18}{\sqrt{x^2 - 2x + 17}} \, dx$ 3