AU–14

B.Sc. (Part-I) Semester-I Examination

(New Course)

1S : MATHEMATICS

Paper—II

(Differential & Integral Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives (1 mark each) : 10

(i) If $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$ and f(x) is defined at x = a then which type of discontinuity occurs :

(a) First kind (b) Second kind

(c) Removable (d) None of these

(ii) Which of the following function is continuous at origin ?

(a) $f(x) = \cos(1/x)$, when $x \neq 0$ and $f(0) = 0$

(b) $f(x) = \sin(1/x)$, when $x \neq 0$ and $f(0) = 0$

(c) $f(x) = x + \sin(1/x)$, when $x \neq 0$ and $f(0) = 1$

(d) $f(x) = x \cdot \sin(1/x)$, when $x \neq 0$ and $f(0) = 1$

(iii) If $y = e^{-3x}$ then $y_{11} = ?$

(a) $-3^{11}e^{-3x}$ (b) $3^{11}e^{-3x}$

(c) $-e^{-3x}$ (d) None of these

(iv) If f(x) is defined and continuous on [a, b]; derivable on (a, b) then there exist at least one point $c \in (a, b)$ such that $f(b) - f(a) = (b - a) f'(c)$ which is the statement of :

(a) Lagranges mean value theorem (b) Rolle's theorem

(c) Cauchy's mean value theorem (d) Intermediate value theorem

(v) If $f(x) = x^2 + x - 6$; $x \in [-3, 2]$ then the value of 'c' by Rolle's theorem is :

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) 1

(vi) For $f(x) = x^2$; $g(x) = x^3$ in [1, 3] then the value of 'c' by Cauchy's mean value theorem is :

(a) $\frac{6}{13}$ (b) $\frac{13}{6}$

(c) 0 (d) 1

(vii) If $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots$ then f(x) is :

(a) $\log(1 + x)$ (b) $\sin x$

(c) $\cos x$ (d) $\tan^{-1}x$

(viii) The value of $\lim_{x \to 0} (x^x)$ is :

(a) e (b) 1/e

(c) 0 (d) 1

(ix) What is the value of $\lim_{x \to \infty} \left[\frac{2^{n+1} + 3^{n+1}}{2^n + 3^n} \right]$?

(a) $\frac{1}{3}$ (b) 1

(c) 3 (d) 0

(x) The area bounded by the curve $x = g(y)$; y-axis and $y = a$, $y = b$ is :

(a) $\int_a^b y\, dx$ (b) $\int_a^b x\, dy$

(c) $\int_a^b y^2\, dx$ (d) None of these

UNIT—I

2. (a) Prove that limit of function, if it exist, then it is unique. 4

(b) Show that the function $f(x) = x \cdot \sin\left(\frac{1}{x}\right); x \neq 0$ is continuous at x = 0. 3

$= 0;$ otherwise

(c) If f(x) is defined and continuous in [a, b] then prove that f(x) attain every value between its bounds. 3

3. (d) Prove that limit of product of two functions is equal to the product of their limits. 4

(e) Show that the function $f(x) = (1+2x)^{1/x}; x \neq 0$ is continuous at x = 0. 3

$= e^2 \quad ; x = 0$

(f) Using $\in-\delta$ definition, prove that :

$$\lim_{x\to 3}\left(\frac{1}{x}\right) = \frac{1}{3}.$$ 3

UNIT—II

4. (a) State and prove Leibnitz theorem. 5

(b) Evaluate $\lim_{x\to 0}\left[\frac{e^x - e^{-x} - 2\log(1+x)}{x \cdot \sin x}\right]$. 3

(c) Find y_n, if $y = (ax + b)^{-1}$. 2

5. (d) If $y = (x + \sqrt{x^2 - 1})^m$ then prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 4

(e) If $y = \frac{x^3}{x^2 - 1}$ then find (y_n) at x = 0. 3

(f) Prove that $\lim_{x\to\infty}\left[\frac{\pi}{2} - \tan^{-1}x\right]^{1/x} = 1$. 3

UNIT—III

6. (a) State and prove Lagranges mean value theorem. 5

(b) Verify Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in [a, b]. 3

(c) Expand e^x upto first four terms at x = 0. 2

7. (d) If f(x) and g(x) are continuous real valued functions on [a, b]; which are differentiable in (a, b) then prove that there exist at least one point 'c' in (a, b) such that :

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}\text{; where } g(a) \neq g(b).$$ 4

(e) Expand $2x^3 + 7x^2 + x - 1$ in powers of (x − 2). 3

(f) Verify Rolle's theorem for $f(x)=\log\left[\frac{x^2+ab}{(a+b)x}\right]$ in [a, b]; $x \neq 0$. 3

UNIT—IV

8. (a) State and prove Euler's theorem for function of two variables. 4

(b) If $u=\frac{x^2+y^2}{x+y}$ then prove that $\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2=4\left(1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)$. 3

(c) If $u=\frac{1}{\sqrt{x^2+y^2+z^2}}; x^2+y^2+z^2 \neq 0$; then show that $u_{xx} + u_{yy} + u_{zz} = 0$. 3

9. (d) If F(u) be a homogeneous function of degree 'n' in x and y, where u is a function of x, y then prove that :

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=n\frac{F(u)}{F'(u)}$ and

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=G(u)[G'(u)-1]$;

where G(u) = nF(u) / F′(u). 5

(e) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z} \text{ and}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{9}{(x + y + z)^2}.$$ 3

(f) If $f(x, y) = 2x^3y^2 - 3xy^2 + x - 2y$ then prove that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$. 2

UNIT—V

10. (a) Prove that :

$$\int \cos^m x \cdot \sin^n x \, dx = \frac{\cos^{m-1} x \cdot \sin^{n+1} x}{m + n} + \frac{m - 1}{m + 1} \int \cos^{m-2} x \cdot \sin^n x \, dx.$$ 4

(b) Evaluate :

$$\int_0^1 \frac{1 - 4x + 2x^2}{\sqrt{2x - x^2}} dx.$$ 3

(c) Prove that the area of an ellipse $b^2x^2 + y^2a^2 = a^2b^2$ is πab. 3

11. (d) If $\phi(n) = \int_0^{\pi/4} \tan^n x \, dx$ then prove that $\phi(n) + \phi(n - 2) = \frac{1}{n - 1}$ and hence find the value of $\phi(5)$. 4

(e) Evaluate $\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}} dx$. 3

(f) Find the length of the arc of the curve $y = \log\left(\frac{e^x - 1}{e^x + 1}\right)$ from $x = 1$ to $x = 2$. 3

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