AS–1347

B.Sc. (Part—I) Semester—I Examination

MATHEMATICS

Paper—II

(Differential & Integral Calculus)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives (**1** mark each) :— 10

(i) If the function f(x) is differentiable at $x = x_o$, then it is :

(a) Not defined at $x = x_o$ (b) Continuous at $x = x_o$

(c) Not continuous at $x = x_o$ (d) None of these

(ii) The function f(x) has simple discontinuity if :

(a) $f(x^+)$, $f(x^-)$ do not exist

(b) $f(x)$, $f(x^+)$ $f(x^-)$ exist but not equal

(c) $f(x^+) = f(x^-) \neq f(x)$

(d) $f(x^+) \neq f(x^-)$

(iii) If $y = \sin(ax + b)$ then y_n is :

(a) $a^n \cos(ax + b + \frac{\pi}{2})$ (b) $a^n \sin(ax + b + \frac{\pi}{2})$

(c) $a^n \sin(ax + b + \frac{n\pi}{2})$ (d) $a^n \sin(ax + b - \frac{n\pi}{2})$

(iv) The graph of function $y = f(x)$, $\forall\ x \in [a, b]$ which satisfies all conditions of Rolle's theorem then geometrically, there exists at least one point c on the curve between $x = a$ and $x = b$ at which the tangent to the curve is :

(a) Parallel to y-axis (b) Parallel to x-axis

(c) Perpendicular to x-axis (d) Perpendicular to y-axis

(v) The expansion of the function e^x is :

(a) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+....$ (b) $1-\frac{x^2}{2!}+\frac{x^4}{4!}+....$

(c) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....$ (d) $x-\frac{x^3}{3!}+\frac{x^5}{5!}+....$

(vi) The value of $\int \log x\, dx$ is :

(a) $\log x + k$ (b) $x \log x - x + k$

(c) $x \log x + x + k$ (d) $\log x - x + k$

(vii) The value of $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x^2}}$ is :

(a) $e^{\frac{1}{2}}$ (b) $e^{-\frac{1}{2}}$

(c) 0 (d) 1

(viii) The degree of homogenous function, $f(x, y) = \sqrt{\dfrac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}}$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{12}$ (d) $\frac{1}{2}$

(ix) If $I_n = \int \sec^n dx$ then the reduction formula for I_n is :

(a) $I_n = -\frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}$

(b) $I_n = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}$

(c) $I_n = \frac{1}{n-1} \sec^{n-1} x \tan x - \frac{n-2}{n-1} I_{n-2}$

(d) $I_n = \frac{1}{n+1} \sec^{n+1} x \tan x - \frac{n-2}{n-1} I_{n-2}$

(x) Let f(x) be continuous and non-negative on [a, b]. Then the area A bounded by curve y = f(x), the x-axis and two ordinates x = a, x = b is :

(a) $A = \int_a^b x\,dx$

(b) $A = \int_a^b y\,dx$

(c) $\int_b^a y.dx$

(d) $\int_{-a}^{-b} f(x)\,dx$

UNIT—I

2. (a) Prove that if $\lim_{x \to x_0} f(x)$ exists, then it is unique. 4

(b) Using the $\epsilon-\delta$ definition, show that $\lim_{x \to 2} x^2 = 4$. 3

(c) Show that $f(x) = \frac{1}{1-e^{\frac{1}{x}}}$ has simple discontinuity at x = 0. 3

3. (a) Prove that if f(x) is defined and continuous in [a, b], then it attains its bounds at least once in [a, b]. 4

(b) Using $\epsilon-\delta$ definition, prove that f(x) = sin x is continuous for all real values of x. 3

(c) Prove that $\lim_{x \to 2} f(x) = 7$, where f(x) = 2x + 3, $\forall$ x $\in$ [0, 5]. 3

UNIT—II

4. (a) Prove that : If f(x) is differentiable at $x = x_0$, then it is continuous at $x = x_0$. Is converse of this statement true ? Justify. 4

(b) Find the n^{th} differential coefficient of $\frac{1}{6x^2 - 5x + 1}$. 3

(c) If $y = x^n \log x$, then show that $y_{n+1} = \frac{n!}{x}$. 3

5. (a) State and prove Leibnitz's theorem. 4

(b) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\tan 5x}{\tan x}$. 3

(c) Prove that $\lim_{x \to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right] = -\frac{1}{2}$. 3

UNIT—III

6. (a) State and prove Lagrange's mean value theorem. 4

(b) Verify Cauchy mean value theorem for the functions :

$f(x) = e^x$ and $g(x) = e^{-x}$ in [a, b]. 3

(c) Expand $3x^3 + 4x^2 + 5x - 3$ about the point x = 1 by Taylor's theorem. 3

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7. (a) State and prove Rolle's theorem. 4

(b) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$. 3

(c) By using Lagrange's mean value theorem, show that $1 + x < e^x < 1 + xe^x, \forall x > 0$. 3

UNIT—IV

8. (a) Let F(u) be a homogenous function of degree n in x and y, where u is function of x and y. Then prove that :

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{nF(u)}{F'(u)}=G(u)$ and

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=G(u)[G'(u)-1]$. 4

(b) If $u=\frac{x^2+y^2}{x+y}$, prove that $(u_x - u_y)^2 = 4[1 - u_x - u_y]$. 3

(c) Verify Euler's theorem on homogeneous function for $u=\log\left[\frac{x+y}{x-y}\right]$. 3

9. (a) If $u = F(x - y, y - z, z - x)$, then prove that :

$u_x + u_y + u_z = 0$. 4

(b) If $u=\sin^{-1}\left\{\frac{x^2+y^2}{x+y}\right\}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$. 3

(c) Show that : $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$, if $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$. 3

UNIT—V

10. (a) Prove that $\int \cot^n x \, dx = -\frac{1}{n-1}\cot^{n-1} x - I_{n-2}$. Hence evaluate $\int \cot^5 x \, dx$. 4

(b) Calculate the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 3

(c) Evaluate : $\int \frac{2x^2+3x+7}{\sqrt{x^2+x+1}} dx$. 3

11. (a) Prove that : $\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x \, dx$. 4

(b) Find the area between the curve $y = x^3 - 3x^2 + 2x$ and the x-axis. 3

(c) Find the length of the arc of the parabola $x^2 = 4ay$ from the vertex to an extremity of the latus rectum. 3