**B.Sc. (Part—I) Semester—I Examination**

**MATHEMATICS**

**Paper—II**

**(Differential & Integral Calculus)**

Time : Three Hours] [Maximum Marks : 60

**Note** :—(1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **one** question from each unit.

1. Choose the correct alternatives (1 mark each) :— 10

(i) Let $f(x) = \sin\frac{1}{x}, \; x \neq 0$

$= 0 \quad , \; x = 0$

Then f(x) has discontinuity of _______ at x = 0.

(a) Type-II (b) Ordinary

(c) Removable (d) None of these

(ii) Let f(x) = [x] = greatest positive integer not greater than x,

then $\lim_{x\to 2} f(x) =$

(a) 0 (b) 1

(c) 2 (d) does not exist

(iii) If $y = (2x - 3)^4$ then $y_3$ =_______.

(a) 192 (b) (2x –3)

(c) 192(2x –3) (d) 0

(iv) A function f(x) has a derivative at $x = x_0$ iff ________.

(a) $f'(x_0^+) = f'(x_0^-)$ (b) $f'(x_0^+) \neq f'(x_0^-)$

(c) $f'(x_0^+) = f'(x_0^-) \neq f'(x_0)$ (d) None of these

(v) If a real function f defined on [a, b] is :

(1) Continuous on [a, b]

(2) Differentiable on (a, b)

then there is at least one point c ∈ (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. It is statement of_____.

(a) Rolle's theorem (b) Lagrange's mean value theorem

(c) Cauchy mean value theorem (d) None of these

(vi) The series of f(x) = sin x is :

(a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....$ (b) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .....$

(c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + .....$ (d) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + .....$

(vii) If $f(x, y) = x^2 + 2xy + y^2$ then $f_{xy}$ = ________.

(a) 1 (b) 2

(c) 3 (d) 4

(viii) If $f(x, y) = \frac{1}{x} + \frac{\log x - \log y + 7}{y}$ then f(x, y) is homogeneous of degree ______.

(a) 1 (b) −1

(c) 2 (d) −2

(ix) Let f(x) be continuous and non-negative on [a, b]. Then the area A bounded by the curve y = f(x), the x-axis and two ordinates x = a, x = b is A = ________.

(a) $\int_b^a y\,dx$ (b) $\int_a^b y\,dx$

(c) $\int_a^b x\,dy$ (d) $\int_b^a x\,dy$

(x) The process of finding the length of arc of a curve by definite integral is known as :

(a) Quadrature (b) Unification

(c) Rectification (d) None of these

## UNIT—I

2. (a) If $\lim_{x \to x_0} f(x) = \ell$, then f is bounded on some deleted neighbourhood of $x_0$, prove this. 3

(b) Show that $\lim_{x \to 1} \frac{2x^3 - x^2 - 8x + 7}{x - 1} = -4$. 3

(c) Discuss the continuity of the function $f(x) = (x - a) \sin \frac{1}{(x - a)}, \quad x \neq a$

$= 0 \quad , \quad x = a.$

at point x = a. 4

3. (p) If $\lim_{x \to x_0} f(x)$ exists, then it is unique. Prove this. 4

(q) Show that $\lim_{x \to 0} f(x)$ does not exist, if $f(x) = \begin{cases} |x|/x & , x \neq 0 \\ 0 & , x = 0 \end{cases}$. 3

(r) Let $f(x) = \begin{cases} \frac{\sin}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, show that f(x) has removable discontinuity at x = 0. 3

## UNIT—II

4. (a) Show that $f(x) = x^2$ is differentiable in $0 \leq x \leq 2$. 3

(b) Find $y_n$ for $y = \tan^{-1}\left(\frac{x}{a}\right)$. 3

(c) If $y = x^n \cdot \log x$, then show that $y_{n+1} = \frac{n!}{x}$. 4

5. (P) Prove that $\lim_{x\to 1}\left[\frac{1}{\log x}-\frac{x}{x-1}\right]=-\frac{1}{2}$. 3

(q) If $y = \cos x \cdot \cos 2x \cdot \cos 3x$, find $y_n$. 3

(r) If $y = e^{a\sin^{-1}x}$, prove that :
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. 4

## UNIT—III

6. (a) If f and g are continuous real functions on [a, b] which are differentiable in (a, b), then there is a point $c\in(a, b)$ such that $\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$, where $g(a)\neq g(b)$ and $f'(x)$, $g'(x)$ are not simultaneously zero. 4

(b) Verify Lagrange's mean value theorem for $f(x) = \log x$ in $[1, e]$. 3

(c) Expand sin x in power of $(x - \frac{1}{2}\pi)$. 3

7. (p) Verify the truth of Rolle's theorem for $f(x) = x^2 + x - 6$ in $[-3, 2]$. 4

(q) Expand $\tan^{-1}x$ in powers of $\left(x-\frac{\pi}{4}\right)$. 3

(r) If f is differentiable on (a, b) and $f'(x) \geq 0$, $\forall\, x\in(a, b)$ then prove that f is monotone increasing on (a, b). 3

## UNIT—IV

8. (a) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2\neq 0$, show that $u_{xx} + u_{yy} + u_{zz} = 0$. 3

(b) If $u = f(x, y)$ is a homogeneous differentiable function of degree n in x, y then $xu_x + yu_y = nu$. Prove this. 4

(c) If $Z=f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then show that :

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2.$$ 3

9. (p) If $u = f(x + ay) + g(x - ay)$, show that $u_{yy} = a^2u_{xx}$. 3

(q) If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, then show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{2}\right)$. 4

(r) If $z = f(x^2 - y^2)$, show that $yz_x + xz_y = 0$. 3

## UNIT—V

10. (a) Find the value $\int_0^1 \frac{1-4x+2x^2}{\sqrt{2x-x^2}}dx.$ 3

(b) Prove that $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots\cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$, n is even.

$= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots\cdots \frac{2}{3}$, n is odd. 4

(c) Calculate the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ 3

11. (p) If $I_n = \int \sin^n x\, dx$ then $I_n = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$. 4

(q) Find the length of the arc of the equiangular spiral $r = ae^{\theta \cot\alpha}$ between the points for which the radii vectors are $r_1$ and $r_2$. 3

(r) Integrate $\int \frac{x^2+2x+3}{\sqrt{x^2+x+1}}dx.$ 3