B.Sc. (Part—I) (Semester—I) Examination
MATHEMATICS
(Algebra & Trigonometry)
Paper—I

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question **ONE** is compulsory. Attempt once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :—

(1) Which **one** of the following statements is true :— 10

(a) $\cosh(x + iy) = \cosh x \cdot \cos y + i\sinh x \cdot \sin y$

(b) $\cosh(x + iy) = \cos x \cos y + i\sin x \sin y$

(c) $\cosh (x + iy) = \cosh x + \cos y - i\sinh x \cdot \sin y$

(d) $\cosh(x + iy) = \cosh x \sin y + i\sinh x \cos y$

(2) What is the value of $\sinh^{-1}x$:

(a) $\log\left[x+\sqrt{x^2+1}\right]$ (b) $\log\left[x+\sqrt{x^2-1}\right]$

(c) $\log\left[x+\sqrt{1-x^2}\right]$ (d) None of these

(3) The value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}$ is ______.

(a) $\pi/2$ (b) $\pi/4$

(c) $\pi/3$ (d) π

(4) Sum of the series $x-\frac{x^2}{2}+\frac{x^3}{3}-......+(-1)^{n+1}\frac{x^n}{n}+.....$; $-1<x<1$ is denoted by ____.

(a) $\log(1 + x)$ (b) $\sinh x$

(c) $\cosh x$ (d) e^x

(5) If $q = 2 + 2i - j + 4k$ then the norm of q is _____.

(a) -5 (b) 5

(c) 1/5 (d) None of these

(6) The inverse of unit quaternion is its ____.

(a) Purely imaginary (b) Purely real

(c) Complex conjugate (d) None of these

(7) If $\alpha + i\beta$ be the root of quadratic polynomial $f(x) = 0$ then its another root is _____.

(a) $\alpha - i\beta$ (b) α

(c) β (d) None of these

(8) If α, β, γ are the roots of the equation $px^3 + qx^2 + rx + s = 0$ then $\Sigma\alpha$ is _____.

(a) $\frac{q}{p}$ (b) $-\frac{q}{p}$

(c) $\frac{r}{p}$ (d) $\frac{s}{p}$

(9) If A and B are the non-signular matrices of order n then ______.

(a) $(AB)^{-1} = AB$ (b) $(AB)^{-1} = A^{-1} \cdot B^{-1}$

(c) $(AB)^{-1} = B^{-1} \cdot A^{-1}$ (b) None of these

(10) 'Every square matrix satisfies its own characteristics equation' is the statement of ______.

(a) Lagrange's MVT (b) De-Moivre's theorem

(c) Cayley-Hamilton theorem (d) Cauchy's MVT

UNIT—I

2. (a) Prove that $\dfrac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta} = \sin\theta + i\cos\theta$.

Hence prove that $\left(1+\sin\frac{\pi}{5}+i\cos\frac{\pi}{5}\right)+i\left(1+\sin\frac{\pi}{5}-i\cos\frac{\pi}{5}\right)=0$. 5

(b) If $\sin(\alpha+i\beta) = x+iy$ then prove that $\dfrac{x^2}{\cosh^2\beta} + \dfrac{y^2}{\sinh^2\beta} = 1$ and $\dfrac{x^2}{\sin^2\alpha} - \dfrac{y^2}{\cos^2\alpha} = 1$ 5

3. (p) Prove that one of the value of :

$(\cos\theta + i\sin\theta)^n$ is $(\cos n\theta + i\sin n\theta)$; when n is negative integer. 5

(q) Separate real and imaginary parts of tan (x + iy). 5

UNIT—II

4. (a) Find the Sum of the series :

$C = 1 + e^{\sin x} \cdot \cos(\cos x) + \frac{1}{2!} e^{2\sin x} \cdot \cos(2\cos x) + e^{3\sin x} \cdot \frac{1}{3!}\cos(3\cos x) + ...$ 5

(b) Prove that $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$. 5

5. (p) Find the sum of the series $\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + ...$ 5

(q) If $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ then prove that

$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + ...+(-1)^{n-1}\frac{1}{2n-1}\tan^{2n-1} x + ...$ 5

UNIT—III

6. (a) Prove that for p, q∈H, N(pq) = N(p) N(q) and N(q*) = N(q). 5

(b) For the quaternion $q = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ and the input vector v = i, compute the output vector w under the action of the operators L_q and L_{q^*}. 5

7. (p) Show that the quaternian product need not be commutative. 5

(q) For any p, q∈H, show that pq = qp if and only if p and q are parallel. 5

UNIT—IV

8. (a) Find the roots of the equation, $8x^3 + 18x^2 - 27x - 27 = 0$, if these roots are in geometric progression. 5

(b) State Descartes rule of sign. Find the nature of the roots of the equation $2x^7 - x^4 + 4x^3 - 5 = 0$. 5

9. (p) Prove that in an equation with real coefficients **complex roots occur** in pairs. 5

(q) Solve the equation $x^4 - 2x^3 - 22x^2 + 62x - 15 = 0$; **given that** $2\sqrt{3}$ is one of the root. 5

UNIT—V

10. (a) Show that if λ is the eigen value of a nonsingular matrix A then λ^{-1} is the eigen value of A^{-1}. 5

(b) Find the eigen values and the corresponding eigen vector for smallest eigen value of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. 5

11. (p) Show that the eigen values of any square matrix A and A' are same. 5

(q) Reduce to canonical form and find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 5