First Semester B. Sc. (Part – I) Examination

(New)

MATHEMATICS

Paper – I

(Algebra and Trigonometry)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **One** is compulsory and attempt it once only.

(2) Attempt **One** question from each unit.

1. Choose the correct alternative :—

(i) If $z = 1 + i\sqrt{3}$, then $|Z|$ is

(a) 0

(b) 1

(c) 2

(d) 3. 1

(ii) The value of $e^{-\pi i}$ is

(a) 0

(b) – 1

(c) i

(d) 3. 1

(iii) If $\theta - n\pi = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta$ ....., then the value of n when θ lies between

$\frac{7\pi}{4}$ and $\frac{9\pi}{4}$ is

(a) $n = 2$

(b) $n = -2$

(c) $n = 3$

(d) $n = -3$. 1

(iv) The series $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$ is called

(a) Gregory's series

(b) Euler's series

(c) Rutherford's series

(d) Machin's series. 1

(v) The norm of quaternion $q = 5 + \overline{2i} - \overline{4j} + \overline{2k}$ is

(a) 4

(b) 5

(c) 6

(d) 7. 1

(vi) The identity quaternion has

(a) both real and vector part zero

(b) both real and vector part one

(c) real part one and vector part zero

(d) none of these. 1

(vii) The equation $(x^2 + 5)^2 = 0$ must have

(a) Two roots

(b) Three roots

(c) Four roots

(d) Five roots. 1

(viii) The equation with integral coefficients having a root $-2 + \sqrt{3}$ is :

(a) $x^2 - 4x + 1 = 0$

(b) $x^2 + 4x + 1 = 0$

(c) $x^2 - 4x - 1 = 0$

(d) $x^2 + 4x - 1 = 0$. 1

(ix) For a symmetric matrix the eigen vectors are

(a) equal

(b) orthogonal

(c) parallel

(d) none of these. 1

(x) Every square matrix A satisfies its own characteristic equation. This is :

(a) De – Moivre's theorem

(b) Euler's theorem

(c) Cayley – Hamilton theorem

(d) None of these. 1

UNIT – I

2. (a) Show that the continued product of four values of $\left(\cos\frac{\pi}{3} - i \sin\frac{\pi}{3}\right)^{3/4}$ is unity. 3

(b) If $\sin(\alpha + i\beta) = x + iy$, prove that

(i) $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$

(ii) $\frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1$ 4

(c) Prove that $\sinh^{-1} x = \log\{x + \sqrt{x^2 + 1}\}$. 3

3. (p) State De - Moivre's theorem. Prove it for negative integers. 1 + 4

(q) Separate in to real and imaginary parts of $\tan^{-1}(x + iy)$. 5

UNIT – II

4. (a) Prove that

$$\frac{\pi}{4} = 4\left[\frac{1}{5} - \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5}\ldots\right] - \left[\frac{1}{23g} - \frac{1}{3}\cdot\frac{1}{23g^3} + \frac{1}{5}\cdot\frac{1}{23g^5}\ldots\right]$$

5

(b) Sum the series

$$a\cos x - \frac{1}{3}a^3\cos(x + 2y) + \frac{1}{5}\cdot a^5\cos(x + 2y) + \ldots\ldots$$ 5

5. (p) Prove that

$$\frac{\pi}{2\sqrt{3}} = \left[1 - \frac{1}{3^2} + \frac{1}{5}\cdot\frac{1}{3^2} - \frac{1}{7}\cdot\frac{1}{3^3} + \ldots\ldots\right]$$

5

(q) Sum the series

$$\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + \ldots\ldots$$

5

UNIT – III

6. (a) If $p = 2 - \overline{i} + \overline{3j} - \overline{4k}$ and $q = 5 + \overline{2i} - \overline{4j} + \overline{3k}$, then find the quaternion product pq. 5

(b) Show that quaternion product is associative. 5

7. (p) Prove that for any quaternion $\overline{p}, \overline{q} \in H$

$$(pq)^* = q^* p^*.$$ 5

(q) Define : Inverse of the quaternion. Show that for any non zero quaternion q.

$$q^{-1} = \frac{q^*}{N^2(q)}$$ 5

UNIT – IV

8. (a) Prove that an equation with real coefficient complex roots occur in pair. 4

(b) Find the condition that the roots of the polynomial equation $x^3 - ax^2 + bx - c = 0$ are in A. P. 3

(c) State Descarte's rule of signs and find the nature of the roots of equation $3x^4 + 12x^2 + 5x - 4 = 0.$ 3

9. (p) Solve the equation $x^3 - 15x - 126 = 0$ by Cardan's method. 5

(q) Solve the equation
$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$ 5

UNIT – V

10. (a) Find the row rank and column–rank of matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$$

2 + 2

(b) Show that if B is an inverse matrix of the same order as A, then the matrices A and $B^{-1}AB$ have the same characteristic roots. 3

(c) Verify Cayley – Hamilton theorem for the matrix.

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

3

11. (p) Find the eigen values and eigen vectors corresponding to the highest eigen value of

matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

5

(q) Show that the eigen values of a Hermitian matrix are all real 5