**AU–13**

**B.Sc. (Part-I) Semester-I Examination**

**MATHEMATICS**

**(New Course)**

**(Algebra and Trigonometry)**

**Paper—I**

Time : Three Hours] [Maximum Marks : 60

**Note :—** (1) Question No. 1 is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The value of $(\cos\theta - i\sin\theta)^{-n}$ is : 1

(a) $\cos n\theta + i\sin n\theta$ (b) $\cos n\theta - i\sin n\theta$

(c) $\sin n\theta + i\cos n\theta$ (d) $\sin n\theta - i\cos n\theta$

(ii) The value of sin(iz) is : 1

(a) sinh z (b) i sinh z

(c) i sin z (d) sin z

(iii) If $x - n\pi = \tan x - \frac{1}{3}\tan^3 x + \ldots\ldots$, then the value of n when x lies between $-\frac{3\pi}{4}$ and $-\frac{5\pi}{4}$ is : 1

(a) 1 (b) –1

(c) 0 (d) None of these

(iv) The value of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$ is : 1

(a) $\pi$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(v) Hamilton product $\vec{i}\,\vec{j}\,\vec{k} = \ldots\ldots\ldots$ 1

(a) 1 (b) −1

(c) 0 (d) None of these

(vi) If f is selection function, $f(2+3\vec{i}+\vec{j}-\vec{k}) = \ldots\ldots\ldots$ 1

(a) 2 (b) 3

(c) 1 (d) −1

(vii) Every equation of degree n has : 1

(a) n roots (b) more than n roots

(c) less than n roots (d) None of these

(viii) The Descartes rule of signs does not tell about the : 1

(a) Positive root of equation (b) Negative root of equation

(c) Zero root of equation (d) None of these

(ix) Elementary transformations : 1

(a) affect the rank of a matrix

(b) do not affect the rank of matrix

(c) have the different rank of a matrix

(d) None of these

(x) An n-square matrix A has rank $r < n$ iff : 1

(a) det (A) = 0 (b) det (A) ≠ 0

(c) det (A) = ∞ (d) None of these

## UNIT—I

2. (a) By using DeMoivre's theorem, find all the fourth root of 81. 5

(b) If $\alpha$ and $\beta$ are the roots of the equation $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = z^{n+1} \cdot \cos\frac{n\pi}{3}$. 5

3. (p) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that :

$\cos^2\theta = \pm \sin\alpha$. 5

(q) Separate into real and imaginary parts of $\tan(x + iy)$. 5

## UNIT—II

4. (a) Prove that :

$$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}.$$ 4

(b) Sum the series :

$$\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + \text{..........}$$ 6

5. (p) If $x < \sqrt{2} - 1$ then prove that :

$$2\left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \text{.......}\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^2 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^3.$$ 4

(q) Sum the series :

$$a\cos x - \frac{1}{3}a^3\cos(x+2y) + \frac{1}{5}a^5\cos(x+4y) - \text{.........}$$ 6

## UNIT—III

6. (a) Show that quaternian product need not be commutative. 4

(b) Prove that for $p, q \in H$, $N(pq) = N(p)\,N(q)$ and $N(q^*) = N(q)$. 4

(c) Show that the quaternian product of two vectors $\vec{r}$ and $\vec{s}$ is given by $\vec{r}\vec{s} = \vec{r} \times \vec{s} - \vec{r} \cdot \vec{s}$. 2

7. (p) If $Lq(\vec{v}) = q\,\vec{v}\,q^*$, then prove that $f(Lq(\vec{v})) = 0$ and hence show that $Lq(\vec{v}) \in R^3$. 4

(q) If q is a unit quarterian and $\vec{v} = k\vec{q}$, where $k \in R$, then show that the output vector $\vec{w} = Lq(\vec{v}) = k\vec{q}$. 4

(r) Write the quaternian inverse for $q = a\cos\theta - b\,\vec{u}\sin\theta$. 2

## UNIT—IV

8. (a) Solve by Cardon's method $x^3 - 15x = 126$. 5

(b) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$, roots being in A.P. 5

9. (p) State Descartes' rule of sign. Find the nature of the roots of the equation $2x^7 - x^4 + 4x^3 - 5 = 0$. 1+4

(q) Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$. 5

## UNIT—V

10. (a) Reduce the matrix $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ to the normal form and then find its rank. 5

(b) Prove that the eigenvalues of a Hermitian matrix are all real. 5

11. (p) State Cayley-Hamilton theorem. Verify it for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. 1+4

(q) Find the eigenvalues and the corresponding eigenvectors of the matrix :

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$ 5