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AS–1346

B.Sc. (Part—I) Semester—I Examination
MATHEMATICS (New)
Paper—I
(Algebra and Trigonometry)

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :—

(i) If i, 1 + i are the roots of $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$ then remaining roots are :

(a) – i, 1 – i (b) – i, i – 1

(c) i, 1 – i (d) – i, – 1 – i 1

(ii) The real part of sin (x + iy) is :

(a) sin x · cosh y (b) cos x · sinh y

(c) sin x · sinh y (d) cos x · cosh y 1

(iii) The series $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\ldots\ldots$ is called as :

(a) Euler's series (b) Gregory's series

(c) Rutherford series (d) Geometric series. 1

(iv) If $\theta - n\pi = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \ldots\ldots$ then the values of n when θ lies between $\frac{19\pi}{4}$ & $\frac{21\pi}{4}$ is :

(a) n = 4 (b) n = – 4

(c) n = 5 (d) n = –5 1

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(v) The unit quaternion has :

(a) Both real and vector part one

(b) Both real and vector part zero

(c) real part one and vector part zero

(d) None of these. 1

(vi) The norm of quaternion $q = 2 + 2\vec{i} - \vec{j} + 4\vec{k}$ is :

(a) 5 (b) 4

(c) 2 (d) 9 1

(vii) The number of positive and negative roots of an equation of degree n is found by :

(a) Cardon's method (b) Ferrari's method

(c) Descarte's rule of signs (d) None of these 1

(viii) An equation of four degree is called as :

(a) Linear (b) Quadratic

(c) Cubic (d) Biquadratic 1

(ix) The rank of a zero matrix is :

(a) 1 (b) 0

(c) n (d) None of these 1

(x) If the matrix is n-square identity matrix then its rank is :

(a) n (b) 1

(c) 0 (d) None of these 1

UNIT—I

2. (a) State DeMoivre's theorem and prove it for positive integer. 1+4

(b) Find n, n^{th} roots of unity and show that they form a series in G.P. 5

3. (p) Prove that $\cosh^{-1} x = \log\left[x + \sqrt{x^2 - 1}\right]$. 5

(q) If $\sin(\alpha + i\beta) = x + iy$, then prove that $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$ and $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$. 5

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UNIT—II

4. (a) If $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$, then show that :

$$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x - \ldots\ldots + (-1)^{n-1}\frac{1}{2n-1}\tan^{2n-1} x + \ldots.. \quad 5$$

(b) Sum the series $\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + \ldots..$ 5

5. (p) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$. 5

(q) Sum the series $\cos x - \frac{1}{3!}\cos(x+2y) + \frac{1}{5!}\cos(x+4y) - \ldots\ldots$ 5

UNIT—III

6. (a) Show that quaternion product need not be commutative. 5

(b) Prove that for any p, q ∈ H, (pq)* = q* p* . 5

7. (p) Show that for any p, q ∈ H, pq = qp iff $\vec{p}$ and $\vec{q}$ are parallel. 5

(q) Define inverse of a quaternion. Show that for any nonzero quaternion q, $q^{-1} = \frac{q^*}{N^2(q)}$. 1+4

UNIT—IV

8. (a) Prove that in a polynomial equation with real coefficients, complex roots occur in pairs. 5

(b) Solve the equation $x^3 - 15x = 126$ by Cardon's method. 5

9. (p) State Descarte's rule of signs and show that the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four complex roots. 1+4

(q) Find the condition that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P. 5

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UNIT—V

10. (a) Define a row rank and column rank of a matrix. Show that row rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$ is 2. 1+1+3

(b) State Caley-Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. 1+4

11. (p) Define eigenvalues and eigenvectors of a matrix. If λ is an eigenvalue of matrix A, then show that λ^m is an eigenvalue of the matrix A^m, for any positive integral value of m. 1+1+3

(q) Find the eigenvalues and the corresponding eigenvectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. 5

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