(b) Evaluate $\int x\sqrt{\frac{a-x}{a+x}}.dx.$ 3

(c) Show that '8a' is the length of an arc of the cycloid

$x = a(t - \sin t),\ y = a(1 - \cos t);\ 0 \le t \le 2\pi.$ 3

11. (d) Prove that $\int \cot^n \theta\, d\theta = -\frac{(\cot\theta)^{n-1}}{n-1} - \int \cot^{n-2}\theta\, d\theta$

and hence find $\int \cot^4 \theta\, .\, d\theta.$ 3

(e) Evaluate $\int \frac{x^3+3}{\sqrt{x^2+1}}dx.$ 4

(f) Find the area bounded by the curve $y = \log x$, the x-axis and the ordinates $x = a$, $x = b$. 3

B.Sc. Part–I (Semester–I) Examination

(New Course)

1S : MATHEMATICS–II

(Differential & Integral Calculus)

Time—Three Hours] [Maximum Marks—60

Note:– (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative (1 mark each) : 10

(i) If $f(x) = \sin\left(\frac{1}{x}\right)$; when $x \neq 0$ and $f(0) = 0$ then at $x = 0$; $f(x)$ has discontinuity of type :

(a) Type-first (b) Type-second

(c) Removable (d) Ordinary.

(ii) If $y = \frac{1}{x+a}$ = then y_n is equal to :

(a) $\frac{(-1)^{n-1}.(n-1)!}{(x+a)^n}$ (b) $\frac{(-1)^n.n!a^n}{(x+a)^n}$

(c) $\frac{(-1)^n.n!}{(x+a)^{n+1}}$ (d) None of these.

(iii) For which of the following functions Cauchy's mean value theorem is not applicable :

(a) $f(x) = x^2; g(x) = x^3$ in [1, 3]

(b) $f(x) = e^x; g(x) = e^{-x}$ in [a, b]

(c) $f(x) = 2x^2 + 3x + 1; g(x) = x^2 - 3x + 2$ in [0, 3]

(d) $f(x) = \cos x ; g(x) = \sin x$ in $[0, \pi/2]$

(iv) The Maclaurin's expansion of $f(x) = \cos x$ is :

(a) $1-\frac{x^2}{2!}+\frac{x^4}{4!}+....$ (b) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-....$

(c) $x-\frac{x^2}{2!}+\frac{x^3}{3}-....$ (d) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....$

(v) The value of $\lim_{x\to\infty}\frac{\log\left(1+\frac{1}{x}\right)}{1/x}$ is :

(a) 0 (b) 1

(c) –1 (d) None of these.

(b) If $u = f(x, y, z)$ is a homogeneous and differentiable function of degree 'n', then prove that :

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=nu.$$ 3

(c) Verify Euler's theorem for $u = x^3. e^{-x/y}$. 3

9. (d) If $u=\operatorname{cosec}^{-1}\left(\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}\right)$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left\{\frac{13}{12}+\frac{\tan^2 u}{12}\right\}.$$ 3

(e) Evaluate $\frac{\partial^2 z}{\partial x\partial y}$ at the point $x = y = z$ on the surface $x^x.y^y.z^z = k$; where k is constant. 4

(f) Verify Euler's theorem for $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$. 3

UNIT–V

10. (a) Prove that :

$$\int\sin^m x\cos^n x\,dx=-\frac{(\sin x)^{m-1}.(\cos x)^{n+1}}{m+n}+\frac{m-1}{m+n}\int\sin^{m-2}x.\cos^n x\,dx.$$ 4

UNIT–III

6. (a) If the function f(x) is defined and continuous in [a, b] and differentiable in (a, b), then prove that there is at least one point $c \in (a, b)$ such that

$$f(b) - f(a) = (b - a)\, f'(c).$$ 4

(b) Verify Rolle's theorem for $f(x) = x^2(1 - x^2)$ in [0, 1]. 3

(c) Obtain the expansion of sin x upto first four terms about x = 0. 3

7. (d) State and prove Cauchy's mean value theorem. 4

(e) Verify Lagranges mean value theorem for

$$f(x) = x^3 - x^2 - 5x + 3 \text{ in } [0, 4].$$ 3

(f) Expand $3x^3 + 4x^2 + 5x - 3$ in power of $(x - 1)$. 3

UNIT–IV

8. (a) If $z = \log(x^3 + y^3 - x^2y - xy^2)$, then prove that :

(i) $$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{2}{x+y}$$

(ii) $$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = -\frac{4}{(x+y)^2}.$$ 4

(vi) The area bounded by the curve y = sin x and the x-axis on [0, π] is :

(a) $\frac{1}{2}$ (b) 3

(c) 1 (d) 2

(vii) The value of $\int_0^{\pi/2} \cos^4 x\, dx$ is:

(a) $\frac{3\pi}{16}$ (b) $\frac{\pi}{16}$

(c) $\frac{3\pi}{8}$ (d) 3π

(viii) The value of $\int_0^1 \cosh x\, dx$ is:

(a) $\frac{e^2+1}{2e}$ (b) $\frac{e+1}{2e}$

(c) $\frac{e^2-1}{2}$ (d) $\frac{e^2-1}{2e}$

(ix) The value of 'c' by Lagranges mean value theorem for the function $f(x) = x^2 - 3x$ in [1, 2] is :

(a) 2/3 (b) 3/2

(c) 1/2 (d) None of these.

www.sgbauonline.com

(x) The perimeter of the curve $x^2 + y^2 = 4$ is :

(a) π (b) 4π

(c) $\pi/2$ (d) None of these.

UNIT–I

2. (a) Prove that limit of sum of two functions is equal to sum of their limits. 4

(b) Prove that if f(x) is defined and continuous in [a, b] then it is bounded in [a, b]. 3

(c) Prove that $\lim_{x\to 2} x^2 = 4$; by $\in - \delta$ definition. 3

3. (d) Prove that if f(x) is defined and continuous in [a, b] then it attains its bounds at least once in [a, b]. 5

(e) Using $\in - \delta$ definition, prove that

$$\lim_{x\to 1}\left[\frac{2x^3 - x^2 - 8x + 7}{x-1}\right] = -4$$ 3

(f) Examine the continuity of $f(x) = \frac{x^2-4}{x-2}$; $x \neq 2$

$= 0, \quad x = 0$

at x = 2.

If f(x) is discontinuous then discuss the type of discontinuity. 2

UNIT–II

4. (a) If y = a cos(log x) + b sin(log x) then prove that :

(i) $x^2y_2 + xy_1 + y = 0$ and

(ii) $x^2y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 + 1)\, y_n = 0$. 4

(b) If $y = \tan^{-1}\left(\frac{x}{a}\right)$ then prove that

$$Y_n = \frac{(-1)^{n-1}.(n-1)!}{a^n} \sin^n\theta.\sin(n\theta);$$

where $\theta = \tan^{-1}\left(\frac{a}{x}\right)$. 3

(c) Evaluate $\lim_{x\to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right]$. 3

5. (d) If the functions u and v have derivability upto n^{th} order then prove that :

$(uv)_n = u_n v + nc_1 u_{n-1} v_1 + nc_2 u_{n-2} v_2 + ... + uv_n$. 4

(e) If $y = x^n \log x$ then show that $y_{n+1} = \frac{n!}{x}$. 3

(f) Evaluate $\lim_{x\to 0} (\cos x)^{1/x^2}$. 3