11. (p) Trace the curve $9ay^2 = x(x - 3a)^2$. 4

(q) Find the asymptotes of :

$$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 1.$$ 3

(r) Determine the existence and nature of the double point on the curve $x^2(x - y) + y^2 = 0$. 3

AP–2770

B.Sc. (Part-I) Semester-I (Old) Examination

MATHEMATICS-II

Paper—II

(Calculus)

Time—Three Hours] [Maximum Marks—60

N.B. :— Question No. **1** is compulsory and solve **ONE** question from each unit.

1. Choose the correct alternatives (**1** mark each) : 10

(i) The value of $\lim_{x\to 0} (1+x)^{1/x}$ is :

(a) e

(b) e^2

(c) 0

(d) 1

(ii) The function f(x) is said to be continuous at $x = x_0$ iff :

(a) $\lim_{x\to x_0^+} f(x) \neq \lim_{x\to x_0^-} f(x)$

(b) $\lim_{x\to x_0} f(x) = f(x_0)$

(c) $\lim_{x\to x_0^+} f(x) = \lim_{x\to x^-} f(x)$

(d) None of the above

(iii) If $y = e^{-2x}$ then y_{11} is :

(a) $-2^{11} e^{-2x}$

(b) $2^{11} e^{-2x}$

(c) $-2^{11} e^{2x}$

(d) None of the above

(iv) If y = A sin mx + B cos mx then the value of $y_2 + m^2y$ is equal to :

(a) 0

(b) 1

(c) –1

(d) None of the above

(v) If f(x) is defined on [a, b] and it is continuous on [a, b], derivable on (a, b) then there exist at least one point 'C' in (a, b) such that :

$f(b) - f(a) = (b - a) \cdot f'(c)$ is the statement of :

(a) Lagrange's mean value theorem

(b) Rolle's theorem

(c) Cauchy mean value theorem

(d) None of the above

(vi) The value of $\lim_{x \to 0} x \log x$ is :

(a) 0

(b) 1

(c) –1

(d) None of the above

9. (p) If u = f(x, y) be homogeneous function of degree n then prove that :

(i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ are homogeneous functions of degree 'n – 1' in x, y.

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$. 5

(q) If $x^x y^y z^z = c$ then show that :

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{(x \log ex)} \text{ at } x = y = z.$$ 3

(r) Verify Euler's theorem for the homogeneous function :

$$u = \tan^{-1}\left[\frac{(x^2 + y^2)^{1/2}}{y}\right].$$ 2

UNIT—V

10. (a) Trace the curve $y^2 = ax^3$. 4

(b) Find the asymptotes of the curve :

$x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$. 3

(c) Show that the points of inflection of the curve : $y^2 = (x - a)^2 (x - b)$ lie on the line $3x + a = 4b$. 3

7. (p) If f(x) and g(x) are continuous on [a, b] which are differentiable on (a, b) then prove that there exist at least one point $c \in (a, b)$ such that :

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)};$$

where $g(a) \neq g(b)$ and $g'(c) \neq 0$. 4

(q) Verify Lagrange's mean value theorem for

$f(x) = 2x^2 - 7x + 10$ in [2, 5]. 3

(r) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$. 3

UNIT—IV

8. (a) State and prove Euler's theorem for function of two variables. 1+3

(b) If $u=\frac{x^2+y^2}{x+y}$ then prove that :

$$\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2=4\left(1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right).$$ 4

(c) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, then prove that $xu_x + yu_y = 3$. 2

(vii) The function $f(x, y) = x^4y^{-2} + x^2y^{-4}$ is :

(a) Homogeneous of degree 4

(b) Homogeneous of degree 2

(c) Not homogeneous

(d) None of the above

(viii) Mixed partial derivatives of f(x, y) are equal when f(x, y), f_x, f_y, f_{xy}, f_{yx} are :

(a) continuous

(b) discontinuous

(c) differentiable

(d) None of the above

(ix) The curve is symmetrical about x-axis if the equation :

(a) Contains even power of y

(b) Contains even power of x

(c) Contains odd powers of y

(d) None of the above

(x) A double point of the curve f(x, y) = 0 is said to be node if :

(a) $(f_{xy})^2 - f_{xx} \cdot f_{yy} > 0$

(b) $(f_{xy})^2 - f_{xx} \cdot f_{yy} < 0$

(c) $(f_{xy})^2 - f_{xx} \cdot f_{yy} = 0$

(d) None of the above

UNIT—I

2. (a) By using ϵ-δ definition, prove that limit of sum of two functions is equal to sum of their limits. 4

(b) By using ϵ-δ definition, prove that :

$$\lim_{x\to 2}(2x+3)=7.$$ 3

(c) Prove that $f(x) = x^2$ is continuous at $x = 3$, by ϵ-δ definition. 3

3. (p) If f(x) is continuous on [a, b] then prove that it is bounded on [a, b]. 4

(q) By using ϵ-δ definition, prove that :

$$\lim_{x\to 1}\left(\frac{2x^3-x^2-8x+7}{x-1}\right)=-4.$$ 3

(r) Prove that $f(x)=\sqrt{x-2}$ for $2 \le x \le 4$, then f(x) is continuous in [2, 4]. 3

UNIT—II

4. (a) If $y=(x+\sqrt{1+x^2})^m$ then prove that :

(i) $(1 + x^2)y_2 + xy_1 - m^2y = 0$ and

(ii) $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)_{y_n} = 0$. 4

(b) If $y = \cos x \cdot \cos 2x \cdot \cos 3x$, then find y_n. 3

(c) Evaluate :

$$\lim_{x\to\infty}\left(\frac{3x^2+2x-5}{5x^2-3x+6}\right).$$ 3

5. (p) State and prove Leibnitz theorem. 5

(q) If $y = e^{ax} \cdot \sin bx$ then prove that :

$$y_2 - 2ay_1 + (a^2 + b^2)\, y = 0.$$ 3

(r) Evaluate :

$$\lim_{x\to 0}(\cos x)^{1/x^2}$$ 2

UNIT—III

6. (a) If f(x) is defined on [a, b] and it is continuous on [a, b], derivable on (a, b) then show that there exist at least one point $s \in (a, b)$ such that :

$$f(b) - f(a) = (b - a)\, f'(c).$$ 4

(b) Verify Rolle's theorem for $f(x) = x^2 + x - 6$ in [–3, 2]. 3

(c) Obtain Maclaurin's series for $f(x) = \log(1 + x)$. 3