

AU - 2512

Third Semester B. E. (Electronics and Telecommunication) Examination
(New)

MATHEMATICS - III

Paper - 3 ET 01

(USC - 11824)

P. Pages : 4

Time : Three Hours]

[Max. Marks : 80

- Note : (1) Separate answer book must be used for each section in the subject Geology, Engineering material of civil branch and Separate answer book must be used for Section A and B in Pharmacy and Cosmetic Tech.
(2) Answer **Three** questions from Section A and **Three** questions from Section B.
(3) Assume suitable data wherever necessary.
(4) Use pen of Blue/Black ink/refill only for writing the answer book.
(5) Use of calculator is permitted.

SECTION A

1. (a) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point P(2, -1, 1) along the normal to the surface $xy + yz + zx = 3$ at the point (1, 1, 1) 5
(b) Find div. F and curl.F where $F = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$. 4
(c) Find the Fourier sine transform of $\frac{\bar{e}^{ax}}{x}$. 5

OR

2. (a) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. 4
(b) (i) For a solenoidal vector F. Show that $\text{curl} \cdot \text{curl} \cdot \text{curl} \cdot \text{curl} F = \nabla^4 F$. 3
(ii) If \bar{F}_1 and \bar{F}_2 are irrotational, show that $\bar{F}_1 \times \bar{F}_2$ is solenoidal. 3
(c) Find the Fourier cosine transform of $f(x) = \begin{cases} 1 & 0 \leq x \leq a \\ 0 & x > a \end{cases}$ 4

3. (a) If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ . 7

- (b) Find the Laurent's expansion of

$$f(z) = \frac{7z-2}{z(z+1)(z-2)} \text{ in } 1 < |z+1| < 3.$$

6

OR

4. (a) Prove that $w = \frac{z}{1-z}$ maps the upper half of the z-plane into the upper half of the w-plane. 6

- (b) Evaluate $\oint_c \frac{e^z}{\cos \pi z} dz$ where $c : |z| = 1$

7

5. (a) Find a real root of the equation $x \log_{10} x = 1.2$ by Regula - Falsi method correct to four decimal places. 6

- (b) Solve $\begin{aligned} 10x - 7y + 3z + 5u &= 6, \\ -6x + 8y - z - 4u &= 5, \\ 3x + y + 4z + 11u &= 2 \\ 5x - 9y - 2z + 4u &= 7 \end{aligned}$

by Gauss elimination method. 7

OR

6. (a) Solve by Relaxation method, the equations

$$\begin{aligned} 10x - 2y - 3z &= 205, \\ -2x + 10y - 2z &= 154 \\ -2x - y + 10z &= 120. \end{aligned}$$

6

- (b) Apply Runge-Kutta method to find an approximate value of y when $x = 1.1$ and $x = 1.2$ given that $\frac{dy}{dx} = x^2 + y^2$ with the condition $y = 1.5$ at $x = 1$. 7

SECTION B

7. (a) Solve by variation of parameter

$$(D^2 + D) y = (1 + e^x)^{-1}.$$

6

(b) Solve $\frac{d^4y}{dx^4} + 2 \frac{d^2y}{dx^2} + y = x^2 \cos x.$

7

OR

8. (a) Solve

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + y = \frac{1}{x} \delta m (\log x).$$

7

(b) Evaluate

$$(D^2 + 5D + 6)y = e^{-2x} \sec^2 x (1 + 2 \tan x).$$

6

9. (a) Solve $y_{n+2} - 2y_{n+1} + y_n = n^2 \cdot 2^n$

5

(b) Solve $p+q = \delta m x + \delta m y$

4

(c) Solve $p^2 - pq = 1 - z^2.$

4

OR

10. (a) Solve

$$y_{n+2} + 3y_{n+1} + 2y_n = \delta m \frac{n\pi}{2}.$$

5

(b) Solve $p^2 - q^2 = \frac{x-y}{z}$

4

(c) Solve $(mz - ny)p + (nx - lz)q = ly - mx.$

4

11. (a) If $\bar{f}(s) = \frac{s^2+1}{s^3+3s^2+2s}$ find $L^{-1}\{\bar{f}(s)\}$

4

(b) Evaluate $\int_0^\infty t^3 e^t \delta mt dt$

4

(c) Solve the Differential equation using Laplace transform

$$(D^2 + 4D + 8)y = 1$$

with $y = 0$ and $y' = 1$ at $t = 0$

6

OR

12. (a) Find Inverse Laplace transform by Convolution theorem of

$$\frac{s^2}{(s^2 + a^2)^2}$$

4

(b) Find L. T. of $\frac{d}{dt} \left(\frac{\delta mt}{t} \right)$

4

(c) Solve the equation

$$\frac{d^3y}{dt^3} + 2 \frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$$

where $y = 1$, $\frac{dy}{dt} = 2$, $\frac{d^2y}{dt^2} = 2$ at $t = 0$

6

